MMDSE-3.4

Sl.No. :

Total No. of Pages : 2

III Semester M.Sc. Degree Examination, June - 2025
(CBCS, SLM Scheme)
MATHEMATICS
Number Theory

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Prove that if $a \equiv b \pmod{m}$ and $n|m$, then $a \equiv b \pmod{n}$. Also prove that $gcd(a, m) = gcd(b, m)$.

b) Prove that there is an infinite number of primes.

c) Solve $5x \equiv 2 \pmod{26}$.

[6+5+5]

Q2) a) If p is a prime then prove that $(p-1)! \equiv -1 \pmod{p}$.

b) Find the solution of the system of congruences :

$3x + 4y \equiv 5 \pmod{13}$, $2x + 5y \equiv 7 \pmod{13}$

c) Prove that the odd prime divisors of the integer $n^2 + 1$ are of the form $4k + 1$.

[6+5+5]

Q3) a) Prove that the functions τ and σ are both multiplicative functions.

b) For any positive integer n, prove that $\varphi(n) = n\sum_{d|n} \frac{\mu(d)}{d}$.

[8+8]

Q4) a) Encipher each using affine cipher

$C \equiv 3p + 7 \pmod{26}$; A THING OF BEAUTY IS JOY FOR EVER.

b) Use Euler's summation formula to deduce the following for

$x \geq 2, \sum_{0 \leq n \leq x} \frac{1}{n \log n} = \log(\log x) + B + O\left(\frac{1}{x \log x}\right)$ where B is a constant.

[8+8]

Q5) a) If p is a prime number and $d \mid p-1$ then show that there are exactly $\varphi(d)$ incongruent integers having order d mode.

b) Prove that integer of the form 2^n has no primitive root for $n \geq 3$.

c) Solve the congruence of $x^{15} \equiv 7 \pmod{19}$.

[6+5+5]

Q6) a) If p is an odd prime and a an odd integer, with $gcd(a, p) = 1$, then $\left(\frac{a}{p}\right) = (-1)^{\sum_{k=1}^{\left(\frac{p-1}{2}\right)} \left[\frac{ka}{p}\right]}$.

b) Use quadratic reciprocity to compute $\left(\frac{7}{43}\right)$. Find an integer x such that $x^2 \equiv 7 \pmod{43}$.

[8+8]

Q7) a) Show that if an integer n can be expressed as the sum of two squares in two different ways, then n is composite.

b) Prove that $F_{m+n} = F_{m-1} F_n + F_m F_{n+1}$.

c) Express 3185 as sum of four squares.

[6+5+5]

Q8) a) Prove that the value of any infinite continued fraction is an irrational number.

b) Define periodic confined fractions. If the number x has a periodic simple continued fraction expansion them prove that x is a quadratic irrational.

c) Explain Pell's equation.

[6+5+5]

