MMDSE-3.4

Sl. No.

III Semester M.Sc. (CBCS) Degree Examination, October/November - 2025
MATHEMATICS
Number Theory

Time : 3 Hours **Maximum Marks : 80**

Answer any Five full questions.

1. a) Define Euclidean algorithm. Let a and b be an any two positive integers and r is the remainder when a is divided by b then show that *gcd(a,b) = gcd(b,r).* **(6+6+4)**

b) Show that there is no polynomial f(n) with integral coefficients that will produce primes for all integers n.

c) Find the gcd of 256 and 1166.

2. a) State and prove chinese remainder theorem. **(6+6+4)**

b) If $n \geq 1$ and $gcd\ (a,n) = 1$ then show that $a^{\phi(n)} \equiv 1 (\mathrm{mod}\, n)$.

c) If p is a prime and k > 0 then prove that $\phi(p^k) = p^k - p^{k-1} = p^k\left(1 - \frac{1}{p}\right)$

3. a) If *f* is a multiplicative function and F is defined by F(n) = $\sum_{d/n} f(d)$ then prove that F is also multiplicative. **(5+8+3)**

b) State and prove Möbius inversion formula.

c) For n > 1, the sum of the positive integers less than *n* and relatively prime to *n* is ½ n $\phi(n)$.

4. a) Encipher each using Affine Cipher $C \equiv 3p + 7 (\mathrm{mod}\, 26)$: "A THING OF BEAUTY IS JOY FOR EVER". **(5+6+5)**

b) If *f* has a continuous derivative *f* on the interval [*y, x*], where 0 < y < x, then prove that

$$\sum_{y<n\leq x} f(n) = \int_y^x f(t)\, dt + \int_y^x \left(t - [t] f'(t)\, dt + f(x)([x] - x)\right) - f(y)([y] - y).$$

c) For all $x \geq 1$ then prove that $\sum_{n\leq x} \sigma(n) = \frac{1}{2} G(2) x^2 + 0(x \log x)$.

[P.T.O.

5. a) Show that every odd prime divisor of $n^2 + 1$ is of the form $4K + 1$. **(4+6+6)**

b) If m, n > 2 and gcd (m,n) = 1 then show that there exists no primitive root (mod mn).

c) Solve the linear congruence of $7x \equiv 2 \pmod 9$

6. a) If p is an prime and a an odd integer with gcd (a,p) = 1 then prove that (a/p) = (-1) $\sum_{k=1}^{(P-½)} \left[\frac{ka}{p} \right]$. **(6+3+7)**

b) Evaluate n of Gauss Lemma for $(5/19)$.

c) State and prove Quadratic reciprocity law.

7. a) Show that any integer, of the form 4k+3 cannot be expressed as sum of two squares. **(6+6+4)**

b) Prove that $F_{n+2}^2 - F_n^2 = F_{2n+2}$

c) If the integer m and n are each the sum of four squares then prove that mn is also the sum of four squares.

8. a) Define infinite continued fraction. Show that every irrational number has a unique representation as an infinite continued fraction. **(6+5+5)**

b) Explain Pell's Equation.

c) Write down $x_0 = \sqrt{23}$ irrational number as an infinite simple continued fraction.
