**MMDSE-3.5**

Sl.No. : **Total No. of Pages : 2**

# III Semester M.Sc. Degree Examination, February - 2024
## (SLM Scheme CBCS)
## MATHEMATICS
## q Series and Theory of Partition

**Time : 3 Hours** **Max. Marks : 80**

***Instructions :*** ***1) Answer any five full questions.***
***2) All questions carry equal marks.***

***Q1)*** a) State the prove q-Binomial theorem.

b) Prove that

$$C\{a+(b-c)z\}\,F+(c-a)(c-b)z\,F[c+1]=ac[1-z\,F(a+1)]$$

c) Prove that $\begin{bmatrix} k+\alpha \\ k \end{bmatrix}_q = \dfrac{(q^{\alpha+1};q)_k}{(q;q)_k}$

**[6+6+4]**

***Q2)*** a) State and prove Jackson's transformation formula.

b) Prove that $\displaystyle\sum_{n=0}^{\infty} \frac{(-aq;q)_n q^{\frac{n(n+1)}{2}}}{(q;q)_n} = (-q;q)_\infty (-aq^2;q^2)_\infty$

**[8+8]**

***Q3)*** State Ramanujan's $1\Psi 1$ summation formula. Give the Z.G. Liu proof of the same. **[16]**

***Q4)*** State quintuple product identity. Give the Bailey's proof of the same. **[16]**

***Q5)*** a) If $ab = cd$, then prove that

$$f(a,b)f(c,d) - f(-a,-b)f(-c,-d) = 2af\left(\frac{b}{c},\frac{c}{b}abcd\right)f\left(\frac{b}{d},\frac{d}{b}abcd\right).$$

b) Prove that $f(a,b)\,f(-a,-b) = f(-a^2,-b^2)\,f(-ab,-ab)$.

c) Prove that $\phi^2(q) - \phi^2(-q) = 8q\psi^2(q^4)$.

**[6+6+4]**

***Q6)*** State and prove Ramanujan's cubic continued fraction identity. **[16]**

***Q7)*** a) Prove that the number of partitions of '$n$' with atmost '$m$' parts is equal to the number of partitions of '$n$' with each part less than or equal to 'm'

b) Prove that $\sum_{n=0}^{\infty} p(n)q^n = \frac{1}{(q;q)_\infty}$

**[8+8]**

***Q8)*** a) Prove that $p(7n+5) \equiv 0 \pmod 7$

b) Prove that $\sum_{n=0}^{\infty}(-1)^n$ pod $(qn+8)q^n \equiv \psi^7(q)$.

**[10+6]**

