MMDSE-3.5

Sl.No. : **Total No. of Pages : 2**

III Semester M.Sc. Degree Examination, June/July - 2024
(CBCS, SLM Scheme)
MATHEMATICS
Course-V :q-Series and Theory of Partition

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Prove that the Binomial theorem

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ from q-Binomial theorem.

b) Prove that

$c\{b+(a-c)z\}\mathrm{F} + (c-a)(c-b)z\,\mathrm{F}[c+1] = bc(1-z)\mathrm{F}[b+1]$

c) Prove that $\lim_{q\to\Gamma} \frac{1}{((1-q)x;q)_\infty} = e^x$.

[6+6+4]

Q2) a) State and prove Sear's transformation formula.

b) Prove that,

$${}_2\phi_1[a,b;cq^{-1};z] - {}_2\phi_1[a,b;c;z] = \frac{cz(1-a)(1-b)}{(q-c)(1-c)}\,{}_2\phi_1[aq,bq;cq;z]$$

[8+8]

Q3) State Ramanujan's ${}_1\psi_1$ summation formula Give the G.N. Andrew's proof of the same. **[16]**

Q4) Prove the followig Bailey's ${}_2\psi_2$ transformation formula **[16]**

$$\sum_{n=\infty}^{\infty} \frac{(a;q)_n (b;q)_n}{(c;q)_n (d;q)_n} z^n$$
$$= \frac{(az;q)_\infty (c/b;q)_\infty (a/d;q)_\infty (dq/abz;q)_\infty}{(z;q)_\infty (q/b;q)_\infty (d;q)_\infty (cd/abz;q)_\infty}$$
$$\times \sum_{n=\infty}^{\infty} \frac{(a;q)_n (abz/d;q)_n}{(c;q)_n (az;q)_n} \left(\frac{d}{a}\right)^n .$$

Q5) a) Prove that $f(a,b) - f(-a,-b) = 2af\left(\frac{b}{a}\frac{a}{b}a^4b^4\right)$

b) Prove that $f(a,ab^2) f(b,a^2b) = f(a,b) f(ab,a^3b^3)$.

c) Prove that $\phi(q) + \phi(-q) = 2\phi(q^4)$

[6+6+4]

Q6) If $p := \frac{\phi(-q)}{\phi(q)}$ and $\phi := \frac{\phi(-q^3)}{\phi(q^3)}$, then prove that **[16]**

$$\left(\frac{P}{Q}\right)^2 + \left(\frac{Q}{P}\right)^2 + 6 = 4\left\{PQ + \frac{1}{PQ}\right\}.$$

Q7) a) Prove that the number of partitions of '*n*' into at must '*k*' parts is equal to the number of partitions of '*n*' in which the largest part does not exceed '*k*'.

b) Prove that $(q;q)_\infty = 1 + \sum_{n=1}^{\infty} -\left[q^{\frac{n(3n+1}{2}} - q^{\frac{n(3n-1)}{2}}\right]$

[8+8]

Q8) a) Prove that $p(11n+6) \equiv 0 \pmod{11}$

b) Prove that ped $(9n+7) \equiv 0 \pmod 3$

[10+6]