MMDSE-3.5

Sl.No. : **Total No. of Pages : 2**

III Semester M.Sc. Degree Examination, May/June - 2026
(SLM Scheme, CBCS)
MATHEMATICS
Course - V : q Series and Theory of Partition

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) State and prove q-Binomial theorem.

b) Prove that, $(b-a)\mathrm{F}+a\mathrm{F}(a+1)=b\mathrm{F}(b+1)$.

c) Prove that, Gauss summation formula ${}_2\mathrm{F}_1(a,b;c;1)=\dfrac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$.

[6+6+4]

Q2) a) Prove the following properties of q-binomial coefficient :

i) $\begin{bmatrix} n \\ k \end{bmatrix}_q = \begin{bmatrix} n \\ n-k \end{bmatrix}_q$

ii) $\begin{bmatrix} \alpha \\ k \end{bmatrix}_q = \dfrac{(q^{-\alpha};q)k}{(q;q)_k}\left(-q^{\alpha}\right)^k q^{-\frac{k(k-1)}{2}}$

b) Derive Ramanujan's identity.

[8+8]

Q3) State and prove Ramanujan's ${}_1\Psi_1$ summation formula by Z.G. Liu's proof. **[16]**

Q4) State and prove W.N. Bailey's ${}_2\Psi_2$ bilateral basic hypergeometric series transformation formula. **[16]**

Q5) State and prove quintuple product identity - (By Bailey's proof) **[16]**

Q6) a) If $ab = cd$, then prove the following.

i) $f(a,b)f(c,d)+f(-a,-b)f(-c,-d)=2f(ac,bd)f(ad,bc)$

ii) $f(a,b)f(c,d)-f(-a,-b)f(-c,-d)=2af\left(\frac{b}{c},\frac{c}{b}abcd\right)f\left(\frac{b}{d},\frac{d}{b}abcd\right)$

b) For $|q|<1$, prove that $\varphi(q)=f(q,q)=1+2\sum_{n=1}^{\infty}q^{n^2}=\left(-q;q^2\right)_{\infty}^{2}(q;q)_{\infty}$.

[10+6]

Q7) a) Prove that the numbers of partitions of 'n' with almost 'm' parts is equal to the number of partitions of 'n' with each part less than or equal to 'm'.

b) Prove that, $\sum_{n=0}^{\infty}p(n)q^n=\frac{1}{(q;q)_{\infty}}$.

[8+8]

Q8) Prove that $\varphi^2(q)-\varphi^2(q^5)=4q\frac{f_2^2f_5f_{20}}{f_1f_4}$. **[16]**

