MMDSE-3.5

Sl.No. :

Total No. of Pages : 2

III Semester M.Sc. Degree Examination, June - 2025
(CBCS, SLM Scheme)
MATHEMATICS
Course - V : q Series and Theory of Partition

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Prove that $\left[c-2a+(a+b)z\right]\mathrm{F}+a(1-z)\mathrm{F}(a+1)=(c-a)\mathrm{F}(a-1)$.

b) State and prove q-Binomial theorem.

c) Prove that $\begin{bmatrix} n \\ k \end{bmatrix}_q = \begin{bmatrix} n \\ n-k \end{bmatrix}_q$.

[6+6+4]

Q2) a) State and prove Sear's transformation formula.

b) Prove that ${}_2\phi_1\left(\frac{a}{q},b;c;zq\right)-(1-z)_2\phi_1(a,b;c;z)=\frac{bz\left(1-\frac{c}{b}\right)}{(1-c)}\,{}_2\phi_1(a,b;cq;zq)$.

[8+8]

Q3) State and prove Jacobi's triple product identity by G.N. Andrew's proof. **[16]**

Q4) State and prove Ramanujan's ${}_1\psi_1$ summation formula by Z.G. Liu's proof. **[16]**

Q5) State and prove quintuple product identity by Bailey's proof. **[16]**

Q6) a) Prove that $f\left(x^3q^2,\frac{q}{x^3}\right)-x^{-1}f\left(\frac{q^2}{x^3},qx^3\right)=(q;q)_\infty\frac{f\left(-x^2q,\frac{-1}{x^2}\right)}{f\left(xq,\frac{1}{x}\right)}$.

b) Prove that $\phi(q)-\phi(-q)=4q\psi\left(q^8\right)$.

[8+8]

Q7) a) Let $\mathrm{P}:=\frac{\varphi(-q)}{\varphi(q)}$ and $\mathrm{Q}:=\frac{\varphi\left(-q^5\right)}{\varphi\left(q^5\right)}$, then prove that

$$4\left[(\mathrm{PQ})-\frac{1}{\mathrm{PQ}}\right]=\left(\frac{\mathrm{P}}{\mathrm{Q}}\right)^{\frac{3}{2}}-\left(\frac{\mathrm{Q}}{\mathrm{P}}\right)^{\frac{3}{2}}+5\left[\left(\frac{\mathrm{P}}{\mathrm{Q}}\right)^{\frac{1}{2}}-\left(\frac{\mathrm{Q}}{\mathrm{P}}\right)^{\frac{3}{2}}\right].$$

b) Prove that $\varphi^2(q)+\varphi^2(-q)=2\varphi^2\left(q^2\right)$.

[8+8]

Q8) a) State and prove Schur's theorem.

b) Prove that $p(5n+4)\equiv 0\pmod 5$.

[8+8]

