MMDSE-3.5

Sl. No.

III Semester M.Sc. Degree Examination, October/November - 2025
MATHEMATICS
q-Series and Theory of Partition
(CBCS SLM Scheme)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Answer any Five Full questions.*
2. *All questions carry equal marks.*

1. a) Deduce that $2F_1(a,b;c;z) = \dfrac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a}\,dt$ **(6+6+4)**

b) Prove that $\left[c-2a+(a-b)z\right]F + a(1-z)F(a+1) = (c-a)F(a-1)$

c) Prove that $\lim_{q\to 1^-}\left(-(1-q)x;q\right)_\infty = e^x$

2. a) If $|z|<1,\ |b|<1$ then prove that $2\phi_1(a,\ b;c;\ q,\ z) = \dfrac{(b;q)_\infty (az;q)_\infty}{(c;q)_\infty (z;q)_\infty}\,2\phi_1\left(\dfrac{c}{b},z;az;q,b\right)$.

(8+8)

b) Prove that $\dfrac{\sum_{n=0}^{\infty}\dfrac{q^{\frac{(n^2+3n)}{2}}}{(q;q)_n}}{\sum_{n=0}^{\infty}\dfrac{q^{\frac{(n^2+n)}{2}}}{(q:q)_n}} = \dfrac{1}{1+}\dfrac{q}{1+}\dfrac{q^2}{1+}\dfrac{q^3}{1+}\ldots$

3. State Ramanujan's $|\psi|$ Summation formula. Give the Andrew's and Askey proof of the same. **(16)**

4. State quintuple product identity. Give the Venkata chaliengar's proof of the same. **(16)**

[P.T.O.

5. a) If $ab = cd$, then prove that **(6+6+4)**

$$f(a,b)f(c,d)+f(-a,-b)f(-c,-d)=2f(ac,bd)f(ad,bc)$$

b) Prove that $f(-q,-q)+f(q^2,q^2)=\dfrac{2f^2(q^3,q^5)}{f(q,q^3)}$.

c) Prove that $\phi^4(q)-\phi^4(-q)=16q\Psi^4\left(q^2\right)$

6. Let V(q) denote Ramanujan's cubic continued fraction prove that $\dfrac{1}{v^3(q)}=\dfrac{\psi^4(q)}{q\psi^4\left(q^3\right)}-1$ **.(16)**

7. a) Prove that the number of partitions of 'n' into distinct parts is equal to the number of partitions of 'n' into odd parts. **(8+8)**

b) Prove that $P_{d,e}(n)-P_{d,o}(n)=\begin{cases}(-1)^K, \text{ if } n=\dfrac{k(3k\pm1)}{2}\\ 0, \quad otherwise\end{cases}$

Where $p_{d,e}(n)\{p_{d,o}(n)\}$ denotes the number of partitions into distinct and even {odd} parts.

8. a) Prove that $P(5n+4)\equiv 0 \pmod 5$ **(10+6)**

b) Prove that $\sum_{n=0}^{\infty}(-1)^n\ pod(3n+2)q^n\equiv\psi^5(q)$
