MMDSE - 4.4

Sl.No. :

Total No. of Pages : 2

Fourth Semester M.Sc. Degree Examination, May - 2026
(SLM Scheme) (CBCS)
MATHEMATICS
Graph Theory

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.

2) All questions carry equal marks.

Q1) a) Define the following

i) Graph

ii) Simple graph

iii) Multigraph

iv) Pseudo graph

b) Show that the size of a simple graph of order n cannot exceed ${}^{n}C_{2}$.

c) Prove that the total number of odd degree vertices in a graph is always even.

[6+4+6]

Q2) a) Prove that every u-v walk in graph contains a u-v path.

b) Show that a simple graph G with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

c) For any connected graph G, Prove that rad (G) $\leq$ diam (G) $\leq$ 2 rad (G).

[6+6+4]

Q3) a) Prove that the maximum number of lines among all p point graphs with no triangles is $[p^2/4]$.

b) Define the following

i) Cartesian product of graphs

ii) Composition product of graphs

c) Prove that C_n is self - complementary if and only if $n = 5$.

[5+6+5]

Q4) a) Prove that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.

b) Prove that in a tree there is one and only one path between every pair of vertices.

c) Prove that a tree with n-vertical has (n–1) edges.

[6+4+6]

Q5) a) Define spanning tree. Prove that a graph G = (V.E.) is connected if and only if it has a spanning tree.

b) Prove that every tree has a center consisting of either one point or two adjacent points.

c) Prove that For any graph G, $K(G) \leq \lambda(G) \leq \delta(G)$.

[5+5+6]

Q6) a) Prove that a graph with atleast in vertices is *n*-connected if and only if for any two disjoint sets V_1 and V_2 of *n* vertices each, there exist *n* disjoint paths joining these two sets of vertices.

b) State and proveDirac's theorem.

[8+8]

Q7) a) State and prove Hall's Theorem.

b) Let G be a simple connected graph with *n* vertices, where $n \geq 3$. If the sum of degrees of every pair of non-adjacent vertices is greater than or equal to *n*, then prove that G is Hamilton.

[8+8]

Q8) a) Show that the complete graph of 5 vertices (K_5) and complete bipartite graph $K_{3,3}$ are non-planar.

b) Prove that a connected planar graph with *n* vertices and e edges has e–*n*+2 regions.

[8+8]

