# MMDSE-4.4

Sl.No. :

**Total No. of Pages : 3**

## Fourth Semester M.Sc. Degree Examination, November - 2024

### (SLM Scheme) (CBCS)

### MATHEMATICS

### Graph Theory

**Time : 3 Hours** **Max. Marks : 80**

**Instructions : 1) Answer any five full questions.**

**2) All questions carry equal marks.**

***Q1)*** a) Define a graph, construct a graph G = {(1, 2, 3, 4, 5)}, where $\{(x, y), |x - y| = 1\}$.

b) Show that the size of a simple graph of order $n$ cannot exceed ${}^{n}C_{2}$.

c) Prove that a graph G of order '$p$' and size '$q$' satisfies the condition $\sum_{v=V(G)} \deg(v) = 2q$.

**[4+6+6]**

***Q2)*** a) Define spanning and induced subgraph Distinguish between edge - disjoint and vertex disjoint subgraphs.

b) Prove that a simple graph G with n-vertices and k-components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

c) Prove that a graph is bipartite if and only if all its cycles are even.

**[4+6+6]**

***Q3)*** a) Prove that for any graph G with six points, G or $\overline{G}$ contains a triangle.

b) Define intersection graph. Prove that every graph is an intersection graph.

c) If G is a connected graph with $p > 3$ points. Then $w(G) = q$ if and only if G has no triangle.

**[4+5+7]**

***Q4)*** a) Define :

i) Cartesian product of graphs

ii) Composition product of graphs

iii) Normal product of graphs

iv) Tensor product of graphs

b) Define cut vertices, bridges and blocks of a graph. Prove that every non-trivial connected graph contains at least two vertices that are not cut vertices.

**[8+8]**

***Q5)*** a) Prove that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.

b) Prove that a graph H is the block graph of some graph if and only if every block of H is complete.

c) In a graph G if there is one and only one path between every pair of vertices, prove that G is a tree.

**[6+5+5]**

***Q6)*** a) Define a tree. Prove that in a tree there is one and only one path between every pair of vertices.

b) Show that a tree with n-vertices has ($n$–1) edges.

c) Prove that a sequence $d_1, d_2, d_3, ....., d_n$ ($n \geq 2$) of positive integers is the degree sequence of a tree of order $n$ if and only $\sum_{i=1}^{n} d_i = 2n - 2$.

**[4+4+8]**

***Q7)*** a) Define spanning tree. If G = (V, E) is connected if and only if it has a spanning tree.

b) Prove that the maximum connectivity of a ($p$ . $q$) graph equals the maximum connectivity.

c) State and prove Whitney's theorem.

**[4+4+8]**

***Q8)*** a) State and prove Hall's theorem.

b) Prove that a connected graph G has an Euler circuit if and only if all vertices of G are of even degree.

c) Prove that the complete graph of 5 vertices is non-planar.

**[5+6+5]**

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