MMDSE - 4.4

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, October - 2025
(SLM Scheme) (CBCS)
MATHEMATICS
Graph Theory

Time : 3 Hours **Max. Marks : 80**

Instruction : 1) Answer any five full questions.
2) All questions carry equal marks.

Q1) a) State and prove handshaking property. Hence prove that there is no graph with odd number of odd degree vertices.

b) Show that every *uv* walk in a graph contains a *u*-*v* path.

c) Define a bipartite graph. Show that a graph is bipartite if and only if all its cycles are even.

[6+4+6]

Q2) a) Define eccentricity, radius, diameter and central vertex of a graph. Find radius & diameter for K_n, C_n, and $W_{1,n}$.

b) For any connected graph G, prove that rad (G) ≤ diam (G) ≤ 2 rad (G).

c) Prove that a connected graph of order *n* and size *m* has at least (*n*-*m*) components.

[6+5+5]

Q3) a) Prove that the maximum number of lines among all *p* point graphs with no triangles is $[p^2/4]$.

b) If a simple graph *G* of order *n* is self-complementary, show that either *n* or (*n*-1) must be a multiple of 4.

c) Find Cartesian product, composition product of P_3 and C_4.

[6+6+4]

Q4) a) Prove that every non-trivial connected graph has at least two vertices which are not cut-vertices.

b) Prove that a connected graph with *n*-vertices and (*n*-1) edges is a tree.

c) Let T be a tree of order *k*, ($k \geq 1$) and let G be a non-trivial graph with $\delta(G) \geq k-1$ then prove that T is a sub graph of G.

[6+5+5]

Q5) a) Define spanning tree. Prove that a graph G = (V.E) is connected if and only if it has a spanning tree.

b) Prove that every tree has a center consisting of either one point or two adjacent points.

c) For any graph G prove that $K(G) \leq \lambda(G) \leq \delta(G)$.

[5+5+6]

Q6) a) State and prove Dirac's theorem.

b) State and prove Berge's theorem.

[8+8]

Q7) a) Show that a connected graph G has an Euler circuit if and only if G can be decomposed into edge-disjoint cycles.

b) If G has a Hamilitonian cycle, prove that for each non-empty set $S \subseteq V(G)$, the graph G has at most $|S|$ components.

c) Prove that a complete bipartite graph $K_{3,3}$ is non-planar.

[6+6+4]

Q8) a) If D is the maximum degree of the vertices in a graph G, then prove that the chromatic number of G is less than or equal to (D+1).

b) Prove that every planar graph is 5-colorable.

c) Find the chromatic polynomial of the complete bipartite graph $K_{2,5}$.

[6+5+5]

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