MMDSE-4.6

Sl.No. :

Total No. of Pages : 2

Fourth Semester M.Sc. Degree Examination, May - 2026

(CBCS) (SLM Scheme)

MATHEMATICS

Differential Geometry

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five questions.

Q1) a) For a regular curve in $\mathbb{R}^3$ with no where vanishing Curvature derive the expression for the following
   i) Curvature
   ii) Torsion
   b) Prove that the signed Curvature of a regular curve is the rate at which the tangent vector of a unit - speed plane curve rotates.

[8 + 8]

Q2) a) Derive Serret - Frenet Equations.
   b) Prove that transition maps of a smooth surface are smooth.

[10 + 6]

Q3) a) State and prove four vertex theorems.
   b) State and prove isoperimetric inequality.
   c) Verify whether the surface $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ is smooth surface.

[7 + 7 + 2]

Q4) a) Define a smooth surface. If $f : V \to R$ is a smooth function is an open subset U of R^2 then prove that the graph of f given by graph $(f) = \{(u,v)|(u,v)\in U\}$ is a Smooth Surface.
   b) Prove that mobius band is not orientable.
   c) State and prove Archimede's theorem.

[6 + 5 + 5]

Q5) a) Show that a diffeomorphism $f : S_1 \to S_2$ is an isometry if and only if for any surface patch σ_1 of S_1, the patches σ_1 & fo σ, of S_1 & S_2 have the first fundamental form.

b) State and prove characterisation for an Equiareal diffeomorphism.

c) Determine the area of the part of the paraboloid $z = x^2 + y^2$ with $z \leq 1$ and compare with area of the hemisphere $x^2 + y^2 + z^2 = 1$, $z < 0$

[6 + 5 + 5]

Q6) a) State and prove Meusniers theorem.

b) If $\sigma_u.\sigma_u = E$, $\sigma_u.\sigma_v = F$ and $\sigma_v.\sigma_v = G$ then prove that $\|\sigma_u \times \sigma_v\| = \sqrt{EG - F^2}$. Further, show that area of a surface patch is unchanged by reparametrization.

[8 + 8]

Q7) a) State and prove Euler's theorem.

b) Let K_1 & K_2 be the principal Curvature at a point p of a surface patch σ then prove the following :

i) If $K_1 = K_2 = K$ then $F_2 = KF_1$ & Every tangent vector σ at p is a prinicpal vector.

ii) If $K_1 \neq K_2$ then any two non-zero principal vectors corresponding to K_1 & K_2 respectively are perpendicular.

[8 + 8]

Q8) a) Derive an Expression for the mean Curvature in terms of the Co-efficient of first and second fundamental form Hence obtain mean curvature of the unit sphere.

b) Let $\sigma : U \to R^3$ be a surface, with usual notations,

prove that $\lim_{\delta \to o} \frac{A_n(R_\delta)}{A_r(R_\delta)} = |k|$.

[8+ 8]

