MMDSE-4.6

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, November - 2024
(SLM Scheme) (CBCS)
MATHEMATICS
Differential Geometry

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Define inverse function theorem. Show that the function $f : U \to V$ is a diffeomorphism then at each point $x \in U$, the linear map df_x is an isomorphism. In particular, *dim* U = *dim* V.

b) i) Show that a parameterized curve has a unit-speed reparametrization if and only if it is regular.

ii) Define closed curve. Show that *r* is a regular closed curve, a unit-speed reparametrization of *r* is always closed.

[8 + 8]

Q2) a) Let $r:(a,b) \to \mathbb{R}^2$ be a unit-speed curve, Let $S_o \in (a,b)$ and let Ψ_o be such that $r^{-1}(S_o) = (\cos\psi_o, \sin\psi_o)$

then show that there is a unique smooth function $\psi:(a,b) \to \mathbb{R}$ such that $\psi(S_o) = \phi$ and $r^1(S) = (\cos(\psi(S)), \sin(\psi(S)))$

b) Compute the torsion of the 3 - curve $r(t) = (3t - t^3, 3t^2, 3t + t^3)$.

[8 + 8]

Q3) a) Show that the transition maps of a smooth surface are smooth.

b) Let $F:[o,\pi] \to \mathbb{R}$ be a smooth function such that $F:[o] = F(\pi) = 0$. Then prove that $\int_0^{\pi} \left(\frac{dF}{dt}\right)^2 dt \geq \int_0^{\pi} F(t)^2 dt$ and equality holds if and only if $F(t) = a \sin t$ for all $t \in [o,\pi]$, where a is a constant.

[8 + 8]

Q4) a) Obtain the first fundamental form of the following :

i) Unit Sphere

ii) Torus

b) Let S and $\tilde{S}$ be a smooth map then show that f is a local diffeomorphism if and only if, for all $P \in S$, the linear map $D_P f : T_P S \to T_{f(P)}\tilde{S}$ is invertible.

[8 + 8]

Q5) a) Define isometry. Isometrices preserve intrinsic distance if $f : S_1 \to S_2$ is an isometry of surfaces then show that $\rho(p,q) = \tilde{\rho}(f(P), f(q))$ for any two points p, q in S_1.

b) Show that a diffeomorphism $f : S_1 \to S_2$ is conformal if and only if, for any surface patch σ of S_1 the first fundmental forms of the patches σ of S and $f \cdot \sigma$ of S_2 are proportional.

[6 + 10]

Q6) a) State and prove Archimedes theorem.

b) State and prove Meusniers theorem.

[8 + 8]

Q7) a) State and prove Euler's theorem.

b) Describe the Pseudo sphere curvature for the surface of revolution :

$\sigma(u \cdot v) = (f(u)\cos v, f(u)\sin v, g(u))$

[8 + 8]

Q8) a) Let P be a paint of a flat surface S and assume that P is not an umbilic then show that there is a patch of S containing P that is a ruled surface.

b) Write a short note on the following :

i) Compact Surfaces

ii) The Gauss map

[8 + 8]

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