MMSEC-1

Sl.No. : **Total No. of Pages : 2**

III Semester M.Sc. Degree Examination, June - 2025
(CBCS, SLM Scheme)
MATHEMATICS
Fuzzy Mathematics (SEC - I)

Time : 1½ Hour **Max. Marks : 40**

Instruction : Compulsorily attend all the sections.

SECTION - A

Answer any five questions : **[5×2=10]**

Q1) Define a fuzzy set. Represent the following linguistic concepts as fuzzy sets -

a) Very low temperature
b) Low temperature
c) Medium temperature
d) High temperature
e) Very high temperature in a temperature range 0-100°C

Q2) Define fuzzy intersection and fuzzy union.

Q3) Prove that, $A \subseteq B$ if and only if $A \cap B = A$ and $A \cup B = B$, where $A, B \in F(X)$.

Q4) Let $A, B \in F(X)$, then for any $a, b \in [0,1]$

$${}^{a+}(A \cap B) = ({}^{a+}A) \cap ({}^{a+}B)$$

Q5) If $A_i \in F(X)$, $i \in I$, where I is an index sets, then prove the following property.

$$a\left(\bigcup_{i \in I} A_i\right) \supseteq \bigcup_{i \in I} {}^{a}A_i .$$

Q6) Define increasing generator function and Psuedo-Inverse of the increasing generator.

Q7) Explain the arithmetic operations on Intervals.

SECTION - B

Answer any four questions : **[4×5=20]**

Q8) Prove that a class of real numbers close to 12 is a fuzzy set.

Q9) Define convex sets. Prove that a fuzzy set A on $\mathbb{R}$ is convex iff,

$$\mathrm{A}\left(lx_1+(1-l)x_2\right)\geq \min\left(\mathrm{A}(x_1),\mathrm{A}(x_2)\right),\forall x_1,x_2\in\mathbb{R}$$

Q10) If A and B are fuzzy sets. Then prove the following

a) $\mathrm{A}\subseteq \mathrm{B}$ *iff* ${}^{a}\mathrm{A}\subseteq {}^{a}\mathrm{B}$

b) $\mathrm{A}=\mathrm{B}$ *iff* ${}^{a+}\mathrm{A}={}^{a+}\mathrm{B}$ $\quad \forall a\in 0,1$

Q11) State and prove second decomposition theorem.

Q12) Show that, a function $g_w:0,1\to\mathbb{R}$ defined by $g_w(a)=a^w\,(w>0)$ is an increasing generator and obtain the fuzzy complement generated by g_w.

Q13) Obtain a t-norm generated by,

$$g(a)=\frac{a}{2}\quad ;\ a\neq 1$$

$$g(a)=1\quad ;\ a=1$$

and $i(a,b)=a\cdot b$

SECTION - C

Answer any one questions : **[1×10=10]**

Q14) For any $\mathrm{A}\in\mathrm{F}(\mathrm{X})$. Prove the following conditions holds -

a) ${}^{a}\mathrm{A}=\underset{b<a}{\mathrm{I}}{}^{b}\mathrm{A}=\underset{b<a}{\mathrm{I}}{}^{b+}\mathrm{A}\ \ a\in 0,1$

b) ${}^{a+}\mathrm{A}=\underset{a<b}{\mathrm{U}}{}^{b}\mathrm{A}=\underset{a<b}{\mathrm{U}}{}^{b+}\mathrm{A}\ \ a\in 0,1$

Q15) Let $\mathrm{C}:0,1\to 0,1$ be a function then prove that, C is involutive fuzzy complement if and only if there exists a continuous function $g:0,1\to\mathbb{R}$ which is a increasing generator and $\mathrm{C}(a)=g(-1)\left(g(1)-g(a)\right)$.

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