DECEMBER 2022 **U/ID 46502/URC1A**

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Prove that $\frac{e-1}{e+1} = \frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots}$.

2. Write Newton's backward difference interpolation formula.

3. Define symmetric matrix and give an example.

4. State Cayley Hamilton theorem.

5. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$, find the value of $\Sigma \frac{1}{\alpha\beta}$.

6. Write the formula for finding a real root of the given equation by Newton's method.

7. Show that $\cosh^2 x - \sinh^2 x = 1$.

8. Find Log $(1+i)$.

9. If $x = r\cos\theta, y = r\sin\theta,$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

10. State Leibnitz theorem.

11. Show that $1 - \nabla = E^{-1}$.

12. Show that the matrix $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

SECTION B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions

13. Using Newton's formula, find the value of y when $x = 27$, from the following data.

x	10	15	20	25	30
y	35.4	32.2	29.1	26.0	23.1

14. Show that the matrix $\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix}$ is orthogonal.

15. Solve $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0,$ given that $1 + i$ is a root.

16. Prove that $\sinh^{-1} x = \log\left(x + \sqrt{1 + x^2}\right)$.

17. If $y = \sin\left(m \sin^{-1} x\right)$, then prove that

$$\left(1 - x^2\right) y_{n+2} - (2n+1) x y_{n+1} - \left(n^2 - m^2\right) y_n = 0.$$

18. Increase the roots of the equation

$x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ by 4 and hence solve the equation.

19. Find the radius of curvature of the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ at any point θ on it.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. Sum the series $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!}$.

21. Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$.

22. Solve: $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

23. Prove that $2^8 \cos^5\theta \sin^4\theta = \cos 9\theta + \cos 7\theta$

$$- 4\cos 5\theta - 4\cos 3\theta + 6\cos\theta.$$

24. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$.

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