DECEMBER 2023 **U/ID 46502/URC1A**

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Prove that $\frac{e^2+1}{e^2-1} = \frac{1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots}$.

2. Write Newton's forward difference interpolation ion formula.

3. Show that $E\nabla = \Delta$.

4. Define Hermitian matrix.

5. Define unitary matrix and give an example.

6. Show that the matrix $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & cons\theta \end{pmatrix}$ is orthogonal.

7. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$, find the value of $\Sigma\alpha^2$.

8. Write the formula for finding a real root of the given equation by Newton's method.

9. If $\frac{\sin\theta}{\theta} = \frac{1013}{1014}$, then find the approximate value of θ.

10. Find the Logarithm of $\sqrt{i}$.

11. Write Leibnitz theorem.

12. Find the n-the derivation of a^x.

SECTION B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Using Lagrange's formula, find log10 401 from the following table.

x	400	404	405	407
$\log_{10} x$	2.6021	2.6064	2.6075	2.6096

14. Express $\begin{pmatrix} 2 & 4 & 8 \\ 6 & 2 & 8 \\ 2 & 2 & 2 \end{pmatrix}$ as the sum of a symmetric and skew symmetric matrices.

15. Solve $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + i$ is a root.

16. Increase the roots of the equation

$x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$

by 4 and hence the solve the equation.

17. If $\sin(A + iB) = x + iy$, then prove that

(a) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ and

(b) $\frac{x^2}{\cosh^2 A} + \frac{y^2}{\sinh^2 A} = 1$

18. If $y = a\cos(\log x) + b\sin(\log x)$, then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

19. Find the radius of curvature at any point on the cardioid $r = a(1 + \cos\theta)$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions

20. Find the sum to infinity of the series

$\frac{7}{72} + \frac{7 \cdot 28}{72 \cdot 96} + \frac{7 \cdot 28 \cdot 49}{72 \cdot 96 \cdot 120} + \ldots$

21. Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$.

22. Solve : $x^6 + 2x^5 + 2x^4 - 2x^2 - 2x - 1 = 0$.

23. Prove that $2^6 \cos^3 \theta \sin^4 \theta = \cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos \theta$.

24. Find the maximum value of $f(x, y, z) = x^2 y^2 z^2$, if $x^2 + y^2 + z^2 = a^2$
