**DECEMBER 2024** **U/ID 46502/URC1A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Show that
$$\left(\frac{a-x}{a}\right)+\frac{1}{2}\left(\frac{a-x}{a}\right)^2+\frac{1}{3}\left(\frac{a-x}{a}\right)^3+\ldots=\log a-\log x\,.$$

2. Write Newton's forward difference interpolation formula.

3. Define symmetric matrix and give an example.

4. Define eigen values.

5. Define unitary matrix and give an example.

6. Write the formula for finding a real root of the given equation by Newton's method.

7. If $\frac{\sin\theta}{\theta}=\frac{863}{864}$, then find the approximate value of $\theta$.

8. Find $Log(1+i)$.

9. If $x = r\cos\theta,\ y = r\sin\theta$, then find $\dfrac{\partial(x, y)}{\partial(r, \theta)}$.

10. Write the formula for finding the radius of curvature.

11. Write Leibnitz theorem.

12. Verify whether the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Using Lagrange's formula, find $\log_{10} 401$ from the following table.

| $x$ | 400 | 404 | 405 | 407 |
|---|---|---|---|---|
| $log_{10} x$ | 2.6021 | 2.6064 | 2.6075 | 2.6096 |

14. Express the matrix $\begin{bmatrix} 2 & 4 & 8 \\ 6 & 2 & 8 \\ 2 & 2 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

15. Solve $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + i$ is a root.

16. Prove that $\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$.

17. If $\sin(A + iB) = x + iy$, then prove that

(a) $\dfrac{x^2}{\sin^2 A} - \dfrac{y^2}{\cos^2 A} = 1$ and

(b) $\dfrac{x^2}{\cosh^2 A} + \dfrac{y^2}{\sinh^2 A} = 1$.

18. Increase the roots of the equation

$x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ by 4 and hence solve the equation.

19. Find the radius of curvature at any point on the curve $x = e^t \cos t$, $y = e^t \sin t$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Find the sum to infinity of the series

$$\frac{7}{72} + \frac{7.28}{72.96} + \frac{7.28.49}{72.96.120} + \ldots$$

21. Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{bmatrix}$.

22. Solve : $x^6 + 2x^5 + 2x^4 - 2x^2 - 2x - 1 = 0$.

23. Prove that
$2^6 \cos^3\theta \sin^4\theta = \cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta$.

24. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$.

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