MAY 2022 **U/ID 46502/URC1A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Show that $\dfrac{1+\dfrac{1}{2!}+\dfrac{1}{4!}+\dfrac{1}{6!}+\cdots}{\dfrac{1}{1!}+\dfrac{1}{3!}+\dfrac{1}{5!}+\cdots}=\dfrac{e^2+1}{e^2-1}$.

2. Write Lagrange's interpolation formula.

3. Define symmetric matrix and give an example.

4. Define unitary matrix and give an example.

5. If α,β,γ are the roots of the equation $x^3-6x^2+11x-6=0$, find the value of $\sum\dfrac{1}{\alpha\beta}$.

6. Write the formula for finding a real root of the given equation by Newton's method.

7. If $\dfrac{\sin\theta}{\theta}=\dfrac{1013}{1014}$, then find the approximate value of θ.

8. Find Log $(1+i)$.

9. If $x = r\cos\theta$, $y = r\sin\theta$, then find $\dfrac{\partial(x,y)}{\partial(u,v)}$.

10. Find the n-th derivative of a^x.

11. Show that $1 - \nabla = E^{-1}$.

12. Show that the matrix $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Use Newton's formula to find y when $x = 142$, given that

x :	140	150	160	170	180
y :	3.685	4.854	6.302	8.076	10.225

14. Prove that the following matrix is unitary : $\begin{pmatrix} \dfrac{1+i}{2} & \dfrac{-1+i}{2} \\ \dfrac{1+i}{2} & \dfrac{1-i}{2} \end{pmatrix}$.

15. Solve $2x^3 - 11x^2 + 10x + 8 = 0$, given that one root is twice the other.

16. Express $\frac{\cos 7\theta}{\cos \theta}$ as a polynomial in

(a) $\cos \theta$

(b) $\sin \theta$.

17. If $y^{1/m} + y^{-1/m} = 2x$, then prove that

$(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

18. Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1 and hence solve the equation.

19. Find the radius of curvature at any point of the cardioid $r = a(1 + \cos\theta)$ at any point θ on it. Hence deduce that $\frac{\rho^2}{r}$ is a constant.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that

$$\sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \cdots$$

21. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$.

22. Solve $3x^6 - 16x^5 + 23x^2 + 16x - 3 = 0$.

23. Show that
$2^7 \cos^3\theta \sin^5\theta = \sin 8\theta - 2\sin 6\theta - 2\sin 4\theta + 6\sin 2\theta$.

24. Find the minimum of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$.

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