MAY 2023 **U/ID 46502/URC1A**

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Show that

$$\left(\frac{a-x}{a}\right)+\frac{1}{2}\left(\frac{a-x}{a}\right)^2+\frac{1}{3}\left(\frac{a-x}{a}\right)^3+\ldots.=\log a-\log x\,.$$

2. Write Lagrange's interpolation formula.

3. Define an orthogonal matrix.

4. Define eigen values.

5. If α, β, γ are the roots of the equation $x^3-6x^2+11x-6=0$, fin the value of $\sum\frac{1}{\alpha\beta}$.

6. Define a reciprocal equation.

7. If $\frac{\sin\theta}{\theta}=\frac{1013}{1014}$, then find the approximation value of θ.

8. Find $\text{Log}(1+i)$.

9. Find the n-th derivative of e^{ax}.

10. Write the formula for finding the radius of curvature.

11. Show that $1-\nabla = E^{-1}$.

12. Find the sum of the eigen values of $\begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$.

SECTION B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Using Lagrange's formula, find $\log_{10}$ 401 from the following table.

x	400	404	405	407
$\log_{10} x$	2.6021	2.6064	2.6075	2.6096

14. Show that the matrix $\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix}$ is orthogonal.

15. Solve $x^3 - 9x^2 + 14x + 24 = 0$, given that two of its roots are in the ration 3 : 2.

16. If $\sin(A + iB) = x + iy$, then prove that

(a) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ and

(b) $\frac{x^2}{\cosh^2 A} + \frac{y^2}{\sinh^2 A} = 1$.

17. If $y^{1/m} + y^{-1/m} = 2x$, then prove that

$$(x^2 - 1)y_{n+2} - (2n + 1)\, xy_{n+1} + (n^2 - m^2)y_n = 0.$$

18. Increase the roots of the equation

$$x^4 + 12x^3 + 56x^2 + 120x + 91 = 0$$

by 3 and hence solve the equation.

19. Find the radius of curvature at any point on the curve $x = e^t \cos t,\ y = e^t \sin t$.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Sum the series $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!}$.

21. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$.

22. Solve : $x^6 + 2x^5 + 2x^4 - 2x^2 - 2x - 1 = 0$.

23. Prove that

$2^8 \cos^5\theta \sin^4\theta = \cos 9\theta + \cos 7\theta - 4\cos 5\theta -$

$4\cos 3\theta + 6\cos\theta$

24. Find the maximum and minimum values of the function

$f(x, y) = 2\left(x^2 - y^2\right) - x^4 + y^4$.