MAY 2024 **U/ID 46502/URC1A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Prove that $\frac{e-1}{e+1} = \frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots}{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots}$.

2. Write Lagrange's interpolation formula.

3. Show that $E\nabla = \Delta$.

4. State Cayley Hamilton theorem.

5. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$, find the value of $\sum \frac{1}{\alpha\beta}$.

6. Show that the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

7. Show that $\cosh^2 x - \sinh^2 x = 1$.

8. Find Log $(1 + i)$.

9. If $\frac{\sin\theta}{\theta} = \frac{1013}{1014}$, then find the approximate value of θ.

10. State Leibnitz theorem.

11. Show that $1 - \nabla = E^{-1}$.

12. Find the n-th derivative of a^x.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Using Newton's formula, find the value of y when $x = 27$, from the following data.

x :	10	15	20	25	30
y :	35.4	32.2	29.1	26.0	23.1

14. Show that the matrix $\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ is orthogonal.

15. Solve $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + i$ is a root.

16. Prove that $\sinh^{-1} x = \log\left(x + \sqrt{1 + x^2}\right)$.

17. If $y^{1/m} + y^{-1/m} = 2x$, then prove that

$\left(x^2 - 1\right)y_{n+2} - (2n + 1)xy_{n+1} + \left(n^2 - m^2\right)y_n = 0$.

18. If $y = a\cos(\log x) + b\sin(\log x)$, then prove that

$x^2 y_{n+2} + (2n + 1)xy_{n+1} + \left(n^2 + 1\right)y_n = 0$.

19. Find the radius of curvature of the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ at any point θ on it.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Sum the series $\sum_{n=0}^{\infty} \frac{5n + 1}{(2n + 1)!}$.

21. Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{bmatrix}$.

22. Solve : $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

23. Prove that $2^8 \cos^5 \theta \sin^4 \theta = \cos 9\theta + \cos 7\theta - 4\cos 5\theta - 4\cos 3\theta + 6\cos \theta$.

24. Find the maximum value of $f(x, y, z) = x^2 y^2 z^2$, if $x^2 + y^2 + z^2 = a^2$.
