MAY 2025 **U/ID 46502/URC1A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Prove that $\frac{e^2+1}{e^2-1}=\frac{1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots}$.

2. Write Newton's backward difference interpolation formula.

3. Define an orthogonal matrix.

4. Define Hermitian matrix.

5. If α,β,γ are the roots of the equation $x^3-6x^2+11x-6=0$, find the value of $\sum\frac{1}{\alpha\,\beta}$.

6. Define a reciprocal equation.

7. If α,β,γ are the roots of the equation $x^3-6x^2+11x-6=0$, find the value of $\sum\alpha^2$.

8. Express $\sinh\theta$ interms of e^{θ}.

9. Find the n-th derivative of e^{ax}.

10. Find the Logarithm of $\sqrt{i}$.

11. Show that $1-\nabla=E^{-1}$.

12. Find the sum of the eigen values of $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Using Lagrange's formula, find $\log_{10}$ 401 from the following table.

x	400	404	405	407
$\log_{10} x$	2.6021	2.6064	2.6075	2.6096

14. Show that the matrix $\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ is orthogonal.

15. Solve $x^3-9x^2+14x+24=0$, given that two of its roots are in the ratio 3:2.

16. Increase the roots of the equation

$$x^4+16x^3+83x^2+152x+84=0$$

by 4 and hence solve the equation.

17. If $y=\sin\left(m\sin^{-1}x\right)$, then prove that

$$\left(1-x^2\right)y_{n+2}-(2n+1)x\,y_{n+1}-\left(n^2-m^2\right)y_n=0.$$

18. Increase the roots of the equation

$$x^4+12x^3+56x^2+120x+91=0$$

by 3 and hence solve the equation.

19. Find the radius of curvature at any point on the cardioid $r=a(1+\cos\theta)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Sum the series $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!}$.

21. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.

22. Solve : $x^6+2x^5+2x^4-2x^2-2x-1=0$.

23. Prove that $2^8\cos^5\theta\sin^4\theta=\cos 9\theta+\cos 7\theta-4\cos 5\theta-4\cos 3\theta+6\cos\theta$.

24. Find the maximum and minimum values of the function $f(x, y)=2\left(x^2-y^2\right)-x^4+y^4$.

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