DECEMBER 2022 **U/ID 46504/URC2A**

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the Fourier coefficient a_0 for $f(x) = x \sin x$ in $0 < x < 2\pi$.

2. Evaluate $\int_0^{\pi/2} \sin^6 x \, dx$.

3. Solve $\int x^4 e^x \, dx$.

4. Find the particular integral of $(D^2 + D - 2)y = \sin 2x$.

5. Form the partial differential equation by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$.

6. Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$.

7. Find $L(\cos at)$.

8. Find the Laplace transform of $t^5 - 4t^3 + 3$.

9. Find $L^{-1}\left[\frac{6}{(s-1)^4}\right]$.

10. If $\phi(x, y, z) = x^2yz^3$, find $\nabla\phi$ at the point $(1, 1, 1)$.

11. Prove that the vector $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$ is solenoidal.

12. State Green's theorem.

SECTION B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. If $f(x) = x^2$, $0 < x < \pi$, find the cosine series for $f(x)$.

14. Evaluate $\int_0^{\pi/2} x \sin^5 x \, dx$.

15. Solve $(D^2 + D - 2)y = \sin 2x$.

16. Obtain a complete solution of the partial differential equation $pq + p + q = 0$.

17. Find $L(t\sin^2 t)$.

18. Find $L^{-1}\left[\frac{s+2}{(s^2+4s=5)^2}\right]$.

19. Prove that

$$\nabla\times(\vec{u}\times\vec{v})=(\vec{v}\cdot\nabla)\vec{u}-(\nabla\cdot\vec{u})\vec{v}-[(\vec{u}\cdot\nabla)\vec{v}-(\nabla\cdot\vec{v})\vec{u}]$$

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Obtain the reduction formula for $\int_0^{\pi/2}\cos^n x\ dx$.

21. Solve $(3z-4y)p+(4x-2z)q=2y-3x$.

22. Using Laplace transforms, solve

$$\frac{d^2y}{dt^2}-2\frac{dy}{dt}-3y=\sin t,$$

given that $y=\frac{dy}{dt}=0$ when $t=0$.

23. Show that $\nabla\cdot(r^n\vec{r})=(n+3)r^n$.

24. Verify Gauss's theorem for the function $\vec{F}=x\vec{i}+y\vec{j}+z\vec{k}$ taken over the region bounded by the planes $x=0$, $x=a$; $y=0$, $y=a$; $z=0$, $z=a$.

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