DECEMBER 2023 **U/ID 46504/URC2A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Evaluate $\int x^4 \cos x dx$.

2. Evaluate $\int_0^{\frac{\pi}{2}} \cos^9 x dx$.

3. Solve $\left(D^2 - 9\right)y = 0$.

4. Define general integral.

5. Find the Laplace transform of $\sin^2 t$.

6. If $L\left|f(t)\right| = F(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)ds$.

7. Find the directional derivative of $\phi = xyz - x^2yz$ at the point (1, 2, –1) in the direction of $\vec{i} - \vec{j} - 3\vec{k}$.

8. Check whether the vector $\vec{A} = x^2z^2\vec{i} + xyz^2\vec{j} - xz^3\vec{k}$ is solenoidal.

9. State Stoke's theorem.

10. Find the work done in moving a particle in a force field $\vec{F} = 3xy\,\vec{i} - 5z\,\vec{j} + 10x\,\vec{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$.

11. Eliminate the arbitrary functions f and ϕ from $z = f(x + ay) + \phi(x - ay)$.

12. If $\vec{F} = axy\,\vec{i} + \left(x^2 + 2yz\right)\vec{j} + y^2\,\vec{k}$ is irrotational, then find the value of a.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Obtain a reduction formula for $\int \cos^n x dx$.

14. Solve $\left(D^2 + 4D + 13\right)y = e^{-2x}\cos 3x$.

15. Find $L^{-1}\left[\frac{s}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}\right]$.

16. Show that the vector $\vec{F} = \left(y^2\cos x + z^3\right)\vec{i} + (2y\sin x - 4)\vec{j} + 3xz^2\,\vec{k}$ is irrotational and find its scalar potential.

17. Evaluate $\int_C \left(2xy - x^2\right)dx + \left(x + y^2\right)dy$ where C is the closed curve formed by $x^2 = y$ and $y^2 = x$.

18. Solve $\sqrt{p} + \sqrt{q} = x$.

19. Find $L\left(te^{-2t}\sin 2t\right)$.

PART C — (3 × 10 = 30 marks)
Answer any THREE questions.

20. Obtain the Fourier series for the function $f(x) = \begin{cases} x & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$

21. Solve $(mz - ny)p + (nx + lz)q = ly - mx$.

22. Using Laplace transform, solve

$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = e^{-2t}$ given that $y = 0$, $\frac{dy}{dt} = 1$, when $t = 0$.

23. Prove that $\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla . \vec{A}\right) - \nabla^2 \vec{A}$.

24. Verify Gauss divergence theorem for $\vec{F} = 4x^2 z\,\vec{i} - y^2\,\vec{j} + yz\,\vec{k}$ taken over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

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