DECEMBER 2024 **U/ID 46504/URC2A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Evaluate $\int x^2 \sin x dx$.

2. Evaluate $\int_0^{\frac{\pi}{2}} \cos^9 x dx$.

3. Eliminate the arbitrary functions f and g from $z = f(x+ay) + g(x-ay)$.

4. Solve $\left(D^2 + n^2\right) y = 0$.

5. Define complete integral.

6. Find the Laplace transform of $\sin^2 t$.

7. If $L[f(t)] = F(s)$, then prove that $L\left[e^{-at} f(t)\right] = F(s+a)$.

8. If $\vec{A}$ and $\vec{B}$ are irrotational, then show that $\vec{A} \times \vec{B}$ is solenoidal.

9. Find the directional derivative of $\phi = xy^2$ at the point $(2,1,3)$ in the direction of $\vec{i} + \vec{j} + \vec{k}$.

10. Check whether the vector $\vec{A} = x^2z^2\,\vec{i} + xyz^2\vec{j} - xz^3\vec{k}$ is solenoidal.

11. State Gauss divergence theorem.

12. Find the work done in moving a particle in a force field $\vec{F} = 3xy\,\vec{i} - 5z\,\vec{j} + 10x\,\vec{k}$ along the curve $x = t^2 + 1,\ y = 2t^2, z = t^3$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Obtain a reduction formula for $\int \sin^n x dx$.

14. Solve $\left(D^2 + 4D + 13\right) y = e^{-2x} \cos 3x$.

15. Find $L^{-1}\left[\dfrac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}\right]$.

16. Show that the vector $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational and find its scalar potential.

17. Evaluate $\int_C (2xy - x^2)\,dx + (x + y^2)\,dy$, where C is the closed curve formed by $x^2 = y$ and $y^2 = x$.

18. Solve $z^2\,(p^2 + q^2 + 1) = 1$.

19. Find $L(te^{-t} \sin 4t)$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Obtain the Fourier series for the function

$$f(x) = \begin{cases} x & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$$

21. Solve $x(x^2 - y^2)p + y(x^2 - z^2)\,q = z(y^2 - x^2)$.

22. Using Laplace transform, solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{-x}$ given that $y = 0, \frac{dy}{dx} = 1$, when $x = 0$.

23. Prove that $\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$.

24. Verify Green's theorem for $\int_C \left(3x^2 - 8y^2\right)dx + (4y - 6xy)\,dy$ where C is the boundary of the region enclosed by the parabolas $x^2 = y$ and $y^2 = x$.

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