MAY 2022 **U/ID 46504/URC2A**

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Find the Fourier coefficient a_0 for $f(x) = x \sin x$ in $0 < x < 2\pi$.

2. Define periodic function.

3. Solve $(D^2 - 4D + 3)y = 0$.

4. Find the particular integral of $(D^2 - D - 2)y = e^{2x}$.

5. Form the partial differential equation by eliminating the arbitrary constants from $z = ax + by + ab$.

6. Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2 + z^2)$.

7. Find $L(\sin at)$.

8. Find the Laplace transform of $\cos 4t \sin 2t$.

9. Find $L^{-1}\left(\frac{3}{s^2+9}\right)$.

10. If $\phi(x, y, z) = x^2y - 2y^2z^3$, find $\nabla\phi$ at the point (1, –1, 2).

11. Prove that the vector

$$\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$$

is solenoidal.

12. State Gauss divergence theorem.

SECTION B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Obtain the half range cosine series for the function $f(x) = x$ in $0 \le x \le \pi$.

14. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 8\cos 2x$.

15. Obtain a complete solution of the partial differential equation $pq = 1$.

16. Find $L(te^{-2t}\sin 3t)$.

17. Find $L^{-1}\left[\frac{s^2}{(s^2+4)(s^2+9)}\right]$.

18. If $\vec{F} = x^2 y\,\vec{i} + y^2 z\,\vec{j} + z^2 x\,\vec{k}$, then find curl curl $\vec{F}$.

19. Using Green's theorem, evaluate

$$\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy .$$

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$.

21. Solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = x^3 - 3x^2 + 1$.

22. Using Laplace transform, solve

$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = e^{-2t}$, given $y = 0$, $\frac{dy}{dt} = 1$ when $t = 0$.

23. Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$, when $\vec{F}$ is a vector point function.

24. Verify Stoke's theorem for the function $\vec{F} = y\,\vec{i} + z\,\vec{j} + x\,\vec{k}$ and the surface S is the part of the sphere $x^2 + y^2 + z^2 = 1$ along the xy-plane.

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