MAY 2023 **U/ID 46504/URC2A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Evaluate $\int x^2 \sin x dx$.

2. Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x dx$.

3. Solve $\left(D^2 + 5D + 4\right) y = 0$.

4. Define complete integral.

5. Find the Laplace transform of $\cos^2 t$.

6. If $L[f(t)] = F(s)$, then prove that $L[t f(t)] = -\frac{d}{ds} F(s)$.

7. Find the directional derivative of $\phi = xy^2$ at the point $(2, 1, 3)$ in the direction of $\vec{i} + \vec{j} + \vec{k}$.

8. Find the constant a so that the vector $(x+2y)\vec{i}+(y-2z)\vec{j}+(x+az)\vec{k}$ is solenoidal.

9. State Green's theorem.

10. Find the work done in moving a particle in a force field $\vec{F}=3xy\vec{i}-5z\vec{j}+10x\vec{k}$ along the curve $x=t^2+1,\ y=2t^2, z=t^3$.

11. Eliminate the arbitrary functions f and g from $z=e^x f(y)+e^y g(x)$.

12. Show that $\vec{F}=yz\vec{i}+zx\vec{j}+xy\vec{k}$ is irroational.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$.

14. Solve $\left(D^2-4D+13\right)y=e^{2x}\cos 3x$.

15. Find $L^{-1}\left[\frac{1}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right]$.

16. Show that the vector

$\vec{F} = \left(6xy + z^3\right)\vec{i} + \left(3x^2 - z\right)\vec{j} + \left(3xz^2 - y\right)\vec{k}$ is irrotational and find its scalar potential.

17. Evaluate

$$\iint_S x^2\ dydz + y^2 dzdx + 2z\left(xy - x - y\right) dxdy$$

where S is the surface of the cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

18. Solve $xp + p^2 = q$.

19. Find $L\left(te^{-t}\sin 4t\right)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Obtain the Fourier series for the function $f(x) = \pi - x, 0 < x < 2\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots = \frac{\pi}{4}$.

21. Solve $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$.

22. Using Laplace transform, solve

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{-x}$$

given that $y = 0, \frac{dy}{dx} = 1$, when $x = 0$.

23. Prove that

$$\nabla \times \left(\vec{A} \times \vec{B}\right) = \left(\vec{B}.\nabla\right)\vec{A} - \left(\nabla.\vec{A}\right)\vec{B} - \left[\left(\vec{A}.\nabla\right)\vec{B} - \left(\nabla.\vec{B}\right)\vec{A}\right].$$

24. Verify Stoke's theorem for $\vec{F}\left(x^2 - y^2\right)\vec{i} + 2xy\,\vec{j}$ in the rectangular region in the XOY plane bounded by the lines $x = 0,\ x = a,\ y = 0,\ y = a$.