MAY 2024 **U/ID 46504/URC2A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Evaluate $\int x^4 \ e^x \ xdx$.

2. Evaluate $\int_0^{\frac{\pi}{2}} \sin^8 \ xdx$.

3. Solve $\left(D^2 + n^2\right)y = 0$.

4. Solve $pq = y$.

5. Find the Laplace transform of $\cos^2 2t$.

6. If $L[f(t)] = F(s)$, then prove that $L[e^{-at} \ f(t)] = F(s+a)$.

7. Find the directional derivative of $\phi = yz^2$ at the point $(0, 1, 1)$ in the direction of $\vec{i} + \vec{j} + \vec{k}$.

8. Find the value of m so that the vector $(x + 2y)\vec{i} + (my + 4z)\vec{j} + (5z + 6x)\vec{k}$ is solenoidal.

9. State Gauss divergence theorem.

10. Find the work done by the force $\vec{F} = 3(x + yz)\vec{i} + 7xz^2\vec{j} - 5yz^2\vec{k}$ when it moves a particle from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve $x = t^3,\ y = t^2, z = t$

11. Eliminate the arbitrary functions f and g from $z = x f(y) + y g(x)$

12. If $\vec{A}$ and $\vec{B}$ are irrotational, then show that $\vec{A} \times \vec{B}$ is irrotational.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find the Fourier series for the function $f(x) = x^2$ in $-\pi \le x \le \pi$.

14. Solve $(D^2 - 2D + 2)y = e^x \sin x$.

15. Find $L^{-1}\left[\dfrac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right]$

16. Show that the vector $\vec{F} = (3x^2 + 2y^2 + 1)\vec{i} + (4xy - 3y^2z - 3)\vec{j} + (2 - y^3)\vec{k}$ is irrotational and find its scalar potential.

17. Using Stoke's theorem, evaluate

$\int_C \sin z dx - \cos x dy + \sin y dz$ where C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1, z = 3$.

18. Solve $z^2\left(p^2 + q^2 + 1\right) = 1$.

19. Find $L\left(te^{-4t} \sin 3t\right)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Obtain a reduction formula for $\int \sin^m x \cos^n x dx$.

21. Solve $x\left(z^2 - y^2\right)p + y\left(x^2 - z^2\right)q = z\left(y^2 - x^2\right)$.

22. Using Laplace transform, solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}$$

given that $y(0) = 0,\ y'(0) = 0$

23. Prove that

$$\nabla\left(\vec{A}.\vec{B}\right) = \vec{A}\times\left(\nabla\times\vec{B}\right) + \left(\vec{A}.\nabla\right)\vec{B} + \vec{B}\times\left(\nabla\times\vec{A}\right) + \left(\vec{B}.\nabla\right)\vec{A}$$

24. Verify Green's theorem for $\int_C \left(3x^2 - 8y^2\right)dx + (4y - 6xy)dy$ where C is the boundary of the region enclosed by the parabolas $x^2 = y$ and $y^2 = x$

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