MAY 2025 **U/ID 46504/URC2A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Evaluate $\int x^4 \cos x dx$.

2. Evaluate $\int_0^{\pi/2} \sin^8 x dx$.

3. Solve $\left(D^2 + 5D + 4\right)y = 0$.

4. Define general integral.

5. Find the Laplace transform of $\cos^2 2t$.

6. If $L[f(t)] = F(s)$. then prove that $L[tf(t)] = -\frac{d}{ds}F(s)$.

7. Find the directional derivative of $\phi = xyz - x^2yz$ at the point (1, 2, –1) in the direction of $\vec{i} - \vec{j} - 3\vec{k}$.

8. Find the value of m so that the vector $(x+2y)\vec{i}+(my+4z)\vec{j}+(5z+6x)\vec{k}$ is solenoidal.

9. State Green's theorem.

10. Find the work done in moving a particle in a force field $\vec{F}=3xy\vec{i}-5z\vec{j}-5z\vec{j}+10x\vec{k}$ along the curve $x=t^2+1,\ y=2t^2, z=t^3$.

11. Eliminate the arbitrary functions f and g from $z=xf(y)+yg(x)$.

12. Show that $\vec{F}=yz\vec{i}+zx\vec{j}+xy\vec{k}$ is irrotational.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Obtain a reduction formula for $\int \cos^n x\,dx$.

14. Solve $(D^2-2D+2)y=e^x \sin x$.

15. Find $L^{-1}\left[\frac{1}{(s^2+a^2)(s^2+b^2)}\right]$.

16. Show that the vector $\vec{F} = \left(y^2 \cos x + z^3\right)\vec{i} + \left(2y \sin x - 4\right)\vec{j} + 3xz^2\vec{k}$ is irrotational and find its scalar potential.

17. Using Stoke's theorem, evaluate

$$\int_C \sin z\,dx - \cos x\,dy + \sin y\,dy$$

where C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$.

18. Solve $xp + p^2 = q$.

19. Find $L\left(te^{-2t} \sin 2t\right)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Obtain a reduction formula for $\int \sin^m x \cos^n x\,dx$.

21. Solve $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$.

22. Using Laplace transform, solve

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = e^{-2t}$$

given that $y = 0$, $\frac{dy}{dt} = 1$, when $t = 0$.

23. Prove that

$$\nabla\left(\vec{A}.\vec{B}\right)=\vec{A}\times\left(\nabla\times\vec{B}\right)+\left(\vec{A}.\nabla\right)\vec{B}+\vec{B}\times\left(\nabla\times\vec{A}\right)+\left(\vec{B}.\nabla\right)\vec{A}$$

24. Verify Stoke's theorem for $\vec{F}=\left(x^2-y^2\right)\vec{i}+2xy\vec{j}$ in the rectangular region in the xOy plane bounded by the lines $x=0, x=a, y=0, y=a$.

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