DECEMBER 2022 **U/ID 32501/UCM1A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Determine the quadratic equation $3-2i$ as a root.

2. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find $\sum \alpha^2 \beta$.

3. What is meant by Descartes rule of signs?

4. Solve : $x^3 - 3x + 1 = 0$.

5. Define orthogonal matrix.

6. State Cayley-Hamilton theorem.

7. Evaluate $\varphi(720)$.

8. Define a symmetric matrix and give an example.

9. Find the number of divisors of 480 excluding 1 and 480.

10. Find the highest power of 2 and 5 in 79!.

11. If n is prime, prove that nC_r is divisible by n.

12. State Wilson's theorem.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. If α, β, γ be the roots of $x^3 + px^2 + qx + r = 0$ form the equation whose roots are $\beta\gamma - \alpha^2$, $\alpha\gamma - \beta^2$, $\alpha\beta - \gamma^2$.

14. Discuss the nature of roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.

15. Find the sum to infinity of the series $1 + \frac{3}{4} + \frac{3}{4} \cdot \frac{5}{8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \ldots$

16. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, show that A satisfies the equation $A^2 - 5A - 2I = 0$.

17. Defining $\varphi(N)$. Obtain its value.

18. Establish Fermat's theorem on numbers.

19. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

PART C — (3 × 10 = 30marks)

Answer any THREE questions.

20. Solve $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.

21. Calculate the root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ which lies between 3 and 4 correct to three places of decimals by Horner's method.

22. Find the eigen values and eigen vectors of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

23. Find the highest powers of 2, 5, 7, 11, 13 contained in 1000!

24. Show that $x^5 - x$ is divisible by 30.

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