DECEMBER 2023 **U/ID 32501/UCM1A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. From the third degree equation whose roots are $1 - i$ and 2.

2. If α, β, γ are the roots $x^3 + px^2 + qp + r = 0$ find $\sum \alpha^2$.

3. Change the equation $2x^4 - 3x^3 + 3x^2 - x + 2 = 0$ into another the coefficient of whose highest term will be century.

4. State D'Alembert's Ratio test.

5. Write the expansion of $(1 - x)^{-p/q}$.

6. Sum the series $x^3 + \frac{x^4}{1!} + \frac{x^5}{2!} + \frac{x^6}{3!} + \ldots \infty$.

7. Show that $A = \begin{pmatrix} 1 & 1+i & -2-i \\ 1-i & 2 & 8+i \\ -2+i & 8-i & 0 \end{pmatrix}$ is a Hermitian matrix.

8. State Cayley Hamilton theorem.

9. Find the numbers of divisors of 480.

10. Find $\phi(600)$.

11. State Wilson's theorem.

12. Find the remainder when 2^{1000} is divisible by 17.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign.

14. Remove the fractional coefficients from the equation $x^3 + 1/4\,x^2 - \frac{1}{16}x + \frac{1}{72} = 0$.

15. Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

16. Sum the series $\frac{1}{2.3} + \frac{1}{4.5} + \frac{1}{6.7} + \ldots\infty$.

17. Express the square matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 12 & 4 & -5 \\ -3 & 5 & 6 \end{pmatrix}$ as the sum of symmetric and non symmetric matrix.

18. Find the number of sum of and product of all divisors of 840.

19. How many zeros does 79! have?

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Solve the equation

$$6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$$

21. The equation $x^3 - 3x + 1 = 0$ has a root between 1 and 2. Calculate it to three places of decimals.

22. Find the eigen values and eigen vectors of $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.

23. Find the smallest number with 18 divisors.

24. State and prove Fermat's theorem.

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