DECEMBER 2024 **U/ID 32501/UCM1A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the equation with rational coefficients whose roots are $\sqrt{-1}\ \sqrt{-5}$.

2. If α, β, γ are the roots of $x^3 - 3ax + b = 0$ find $\sum(\alpha - \beta)(\alpha - \gamma)$.

3. Find the equation whose roots are the roots of $x^5 + 6x^4 + 6x^3 - 7x^2 + 2x - 1 = 0$.

4. Find the number of real roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.

5. Sum the series $\frac{1}{3} + \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \ldots\infty$.

6. Evaluate $\log_e\left(\frac{98}{102}\right)$.

7. Define Symmetric matrix.

8. State Cayley–Hamilton Theorem.

9. Find the number of divisors of 288.

10. Find $\phi(729)$.

11. Write two properties of congruence modulo.

12. If x and y are prime to n, then show that $x^{n-1} - y^{n-1}$ is divisible by n.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find the sum of the reciprocals of the roots of the equation $x^5 + x^2 + 10x + 105 = 0$.

14. Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.

15. Dimnish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

16. Solve the equation $x^3 - 9x^2 + 108 = 0$.

17. Find A^{-1} if $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$.

18. If n is not a power of 2, prove that $N = 2^n + 1$ is a composite number.

19. Find the number have it remainder 5,4,3,2 then divided by 6,5,4,3 respectively.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Solve $2x^6 - 9x^5 + 10x^4 - 3x^3 + 10x^2 - 9x + 2 = 0$.

21. Sum the series $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \ldots$

22. Find the eigenvalue and eigen vectors of $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

23. Find the highest power of 3 which divides 3!.

24. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

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