MAY 2022 **U/ID 32501/UCM1A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Form a rational cubic equation which shall have for roots $1, 3, -\sqrt{-2}$.

2. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\alpha^3 + \beta^3 + \gamma^3$.

3. Expand $(1+x)^{p/q}$.

4. Find $\dfrac{e - e^{-1}}{2}$.

5. Define a Hermitian matrix.

6. Show that the product of two orthogonal matrices is orthogonal.

7. Find the number and sum of all the divisors of 360.

8. Define Euler's function $\phi(N)$.

9. Find the value of (a) $\left[\frac{2}{5}\right]$ (b) $\left[4\frac{2}{3}\right]$.

10. Define congruence modulo m.

11. Find the remainder when 2^{1000} is divisible by 17.

12. State Fermat's theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that one of the roots $1 - \sqrt{5}$.

14. Find the roots of the equation $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$.

15. Increase by 7 roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

16. Represent $\begin{bmatrix} 2 & 1 & 4 \\ 8 & -1 & 3 \\ 3 & -5 & 0 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

17. Find the remainder obtained in dividing 2^{46} by 47.

18. Find the higher power of 3 dividing $\lfloor 1000$.

19. Diagonalise the matrix

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}.$$

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Solve $x^4 + 20x^3 - 140x^2 + 430x + 462 = 0$ by removing the second term.

21. Solve $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.

22. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ and hence determine its inverse.

23. Find the product of all the divisors of N.

24. State and prove Wilson's theorem.