MAY 2023 **U/ID 32501/UCM1A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Find the equation with rational coefficients whose roots is $\sqrt{-1} - \sqrt{5}$.

2. Remove the fractional coefficients from the equation $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$.

3. Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

4. Sum the series

$$\frac{1^2}{\lfloor 1} + \frac{1^2 + 2^2}{\lfloor 2} + \frac{1^2 + 2^2 + 3^2}{\lfloor 3} + \ldots\ldots$$

5. Define skew-Hermitian matrix.

6. State Cayley's – Hamilton theorem.

7. Define prime and composite number.

8. Find the smallest number with 18 divisors.

9. Define Congruence modulo n and give an example.

10. Evaluate $\phi(729)$.

11. Define orthogonal matrix.

12. Define similar matrices.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions

13. If α, β, γ are the roots of the equation $x^3 + qx + \gamma = 0$. Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$.

14. Remove the second term of $x^3 - 6x^2 + 10x - 3 = 0$.

15. Find the characteristics equation of the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & -1 \\ -7 & 2 & -3 \end{bmatrix}$.

16. Prove that the sum of the integers less than N and prime to it including unity is $\frac{N\,\phi(N)}{2}$.

17. State and Prove Fermat's theorem.

18. Diagonalise the matrix $\begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$.

19. Show that if $x > 0$

$$\log x = \frac{x-1}{x+1} + \frac{1}{2}\frac{x^2-1}{(x+1)^2} + \frac{1}{3}\frac{x^3-1}{(x+1)^3} + \ldots\ldots$$

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. Solve the equation

$6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

21. Solve $x^3 - 9x^2 + 108 = 0$

22. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

23. If $d_1, d_2, ...d_r$ (including 1 and N) are the divisors of N, show that $\phi(d_1) + \phi(d_2) + ... + \phi(d_r) = N$.

24. State and Prove Wilson's theorem.
