MAY 2024 **U/ID 32501/UCM1A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Form an equation with rational coefficients whose root is $\sqrt{5}+\sqrt{2}$.

2. If α, β, γ are the roots of the equation $ax^3+bx^2+cx+d=0$, find $\alpha+\beta+\gamma$.

3. Remove the fractional coefficients from the equation $x^3-\frac{1}{4}x^2+\frac{1}{3}x-1=0$.

4. Find the number of real roots of the equation $x^3+18x-6=0$.

5. Use Binomial Theorem to find the 7th power of 11

6. Evaluate $\log_e\left(\frac{101}{99}\right)$.

7. Find sum and product of the eigen value of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

8. Show that $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is an orthogonal matrix.

9. Find the sum of all divisors of 360

10. Find $\varphi(210)$

11. If p and q are twin primes, prove that $pq+1$ is a perfect square.

12. State Fermat's theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If α, β, γ are the roots of the $x^3 + px^2 + qx + r = 0$, form the equation whose roots are $\beta+\gamma-2\alpha, \gamma+\alpha-2\beta, \quad \alpha+\beta-2\gamma$.

14. Solve the equation $x^4 + 3x^3 - 3x - 1 = 0$.

15. Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

16. Verify cayley-Hamilton theorem for $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & +1 & -1 \\ 2 & -1 & 2 \end{pmatrix}$.

17. Show that $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is unitary.

18. Find the highest power of 3 which divides 1000!

19. Find the remainder when 2^{46} is divided by 47.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in geometric progression and hence solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$ whose roots are in geometric progression

21. Sum the series $\sum_{n=1}^{\infty} \frac{n^2+3}{n+2} . \frac{x^n}{n!}$

22. Diagonalize the matrix $\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

23. Find the highest powers of 2, 5, 7, 11, 13 contained in 1000!

24. State and prove Wilson's theorem.
