MAY 2025 **U/ID 32501/UCM1A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Form a rational cubic equation which shall have for roots 1, $3 - \sqrt{-2}$.

2. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the value of $\Sigma\alpha^2$.

3. Define Descarte's rule.

4. What is the value of e^x?

5. Define symmetric matrix.

6. State Cayley - Hamilton theorem.

7. What is meant by prime number?

8. Find the number of divisors of 840.

9. Define congruence modulo m.

10. State Fermat's theorem.

11. What is Eigen value?

12. Expand $\log(1+x)$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Remove the fractional coefficients from the equation $x^3 + \frac{m}{4}x^2 - \frac{m^2}{16}x + \frac{m^3}{72} = 0$.

14. Sum the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \ldots\infty$.

15. Prove that the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

16. Find the product of all the divisors of N.

17. With how many zeros does 79! end?

18. Show that the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is diagonalizable.

19. Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

21. Find the real root of the equation $x^3 + 6x - 2 = 0$ by using Horner's method.

22. Verify Cayley - Hamilton theorem of the matrix $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

23. If $d_1, d_2, \ldots d_r$ (including 1 and N) are the divisors of N, then show that $\phi(d_1) + \phi(d_2) + \ldots + \phi(d_r) = N$.

24. State and prove Wilson's theorem.

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