DECEMBER 2022 **U/ID 32502/UCM1B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Expand $\sin^5\theta$ interms of $\sin\theta$.

2. Expand $\cos^4\theta$ interms of $\cos\theta$.

3. Write the expansion of $\tan n\theta$ interms of $\tan\theta$.

4. Write the expansion of $\cos 6\theta$ interms of $\cos\theta$ and $\sin\theta$.

5. Write the expansion of $\sin\theta$ interms of θ.

6. Prove that $\cosh 2x = 1 + 2\sinh^2 x$.

7. If $\tan {}^{x}\!/_{2} = \tanh {}^{x}\!/_{2}$ show that $\cos x \cosh x = 1$.

8. Find $\log(3 + 4i)$.

9. Find principal logarithm of $1 - i$.

10. Expand $\tanh^{-1} x$ interms of logarithmic function.

11. Write the generalization of Gregory's series.

12. Find the sum to n terms of the series $\cos\alpha - \cos(\alpha+\beta) + \cos(\alpha+2\beta) - \ldots\infty$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Express $\cos^4\theta \sin^6\theta$ in a series of cosines of multiples of θ.

14. Prove that

$$\frac{\sin^7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta.$$

15. If $\frac{\tan\theta}{\theta} = \frac{2524}{2523}$ find θ approximately.

16. Separate the real and imaginary part of $\tan^{-1}(x+iy)$.

17. Prove that

$$\cosh 5x = 16\cosh^5 x - 20\cosh^3 x + 5\cosh x$$

18. Show that $\log \dfrac{a+ib}{a-ib} = 2i \tan^{-1} b/a$.

19. Find the sum to n terms of the series

$$\frac{1}{\cos\theta + \cos 3\theta} + \frac{1}{\cos\theta + \cos 5\theta} + \frac{1}{\cos\theta + \cos 7\theta} + \ldots$$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that the equation $\sin 3\theta = a \sin\theta + b\cos\theta + c$ has six roots and that the sum of the six values of θ which satisfy it is equal to an odd multiple of π radians.

21. Evaluate :

(a) $\underset{x\to 0}{Lt} \dfrac{\tan x - \sin x}{\sin^3 x}$;

(b) $\underset{x\to \pi/2}{Lt} (\sec x - \tan x)$.

22. If $u = \log \tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$ show that

(a) $\tanh u/2 = \tan \theta/2$

(b) $\theta = -i \log \tan\left(\pi/4 + i\, u/2\right)$.

23. If $\log \sin(\theta + i\varphi) = A + iB$ show that

(a) $2e^{2A} = \cosh 2\varphi - \cos 2\theta$

(b) $\cos(\theta - B) = e^{2\varphi} \cdot \cos(\theta + B)$.

24. Sum of n terms of the series

$$\tan^{-1}\frac{4}{4\cdot 1^2+3} + \tan^{-1}\frac{4}{4\cdot 2^2+3} + \tan^{-1}\frac{4}{4\cdot 3^2+3} + \ldots$$