DECEMBER 2023 **U/ID 32502/UCM1B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write the expression of $\sin n\theta$ in powers of $\sin\theta$ and $\cos\theta$.

2. Prove that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$.

3. If $x = \cos\theta + i\sin\theta$, what is the value of $\left(x - \frac{1}{x}\right)^n = ?$

4. Prove that $\sin^5\theta = \frac{1}{16}[\sin 5\theta - 5\sin 3\theta + 10\sin\theta]$.

5. Prove that $\cosh^2 x - \sinh^2 x = 1$.

6. Prove that $\tanh^{-1} x = \frac{1}{2}\log\frac{1+x}{1-x}$.

7. Separate into real and imaginary parts of $\sin(x + iy)$.

8. If $\tanh x = \sin\theta$, show that $\cosh x = \sec\theta$.

9. Find $\log(1 + i)$.

10. Find $\log 3$.

11. Write down Gregory's series.

12. Prove that $\frac{1}{2}-\frac{1}{3}.\frac{1}{2^3}+\frac{1}{5}.\frac{1}{2^5}-\ldots=\tan^{-1}\left(\frac{1}{2}\right)$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Expand $\sin^7\theta$ in a series of sines of multiples of θ.

14. Express $\cos 8\theta$ in terms of $\sin\theta$.

15. If $\tan(x+iy)=u+iv$. Prove that $\frac{u}{v}=\frac{\sin 2x}{\sinh 2y}$.

16. Find the sum of the series

$\operatorname{cosec}\theta+\operatorname{cosec}2\theta+\operatorname{cosec}2^2\theta+\ldots+\operatorname{cosec}2^{n-1}\theta$.

17. Reduce $(\alpha+i\beta)^{x+iy}$ to the form $A+iB$.

18. Prove that $\pi=2\sqrt{3}\left\{1-\frac{1}{3^2}+\frac{1}{5}.\frac{1}{3^2}-\frac{1}{7}.\frac{1}{3^3}+\ldots\right\}$.

19. In $\tan A=\tan\alpha\tanh\beta$, $\tan B=\cot\alpha\tanh\beta$. Prove that $\tan(A+B)=\sinh\beta\operatorname{cosec}2\alpha$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Expand $\sin^4\theta\cos^2\theta$ in a series of cosines of multiple of θ.

21. If $\cos\alpha.\cosh\beta=\cos\phi,\sin\alpha\sinh\beta=\sin\phi$. Prove that $\sin\phi=\pm\sin^2\alpha=\pm\sinh^2\beta$.

22. Find the general value of $\log(x+iy)$.

23. Sum to infinity the series

$$\cos\alpha+\frac{1}{2}\cos(\alpha+\beta)+\frac{1}{2}.\frac{3}{4}\cos(\alpha+2\beta)+\ldots$$

24. Separate into real and imaginary parts $\text{at}\tan^{-1}(x+iy)$.

———————