DECEMBER 2024 **U/ID 32502/UCM1B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write the expression of $\sin n\theta$ in powers of $\sin\theta$ and $\cos\theta$.

2. If n is an integer, what is the value of $\sin n\pi$?

3. If $x = \cos\theta + i \sin\theta$, find $\left(x - \frac{1}{x}\right)^n$.

4. Prove that $\cos^5\theta = \frac{1}{16}[\cos 5\theta + 5\cos 3\theta + 10\cos\theta]$.

5. Prove that $\cosh^2 x - \sinh^2 x = 1$.

6. Prove that $\cosh^{-1} x = \log\left[x + \sqrt{x^2 - 1}\right]$.

7. Separate into real and imaginary parts of $\cot(x + iy)$.

8. Prove that $\sin ix = i \sin hx$.

9. Find $\log(1 + i)$.

10. Find $\log(i)$.

11. Write down Gregory's series.

12. Find $\frac{\sin h\alpha}{1!} + \frac{\sin h2\alpha}{2!} + ... + \infty$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Expand $\sin^7 \theta$ in the series of sines of powers of θ .

14. Express $\frac{\sin 6\theta}{\sin \theta}$ interms of $\cos \theta$.

15. Separate real and imaginary parts of $\tan h(1 + i)$.

16. Find the sum of the series

$\text{cosec } \theta + \text{cosec } 2\theta + \text{cosec } 2^2\theta + ... \text{cosec } 2^{n-1}\theta$.

17. Find $\log(1-i)$.

18. Prove that $\pi = 2\sqrt{3}\left\{1 - \frac{1}{3^2} + \frac{1}{5} = \frac{1}{3^2} - \frac{1}{7}\frac{1}{3^3} + \ldots\right\}$

19. If $\cos hu = \sec\theta$, prove that $u = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Expand $\sin^3\theta\cos^5\theta$ in a series of sines of multiples of θ.

21. If $\cos\alpha\cos h\beta = \cos\phi$, $\sin\alpha\sin h\beta = \sin\phi$, prove that $\sin\phi = \pm\sin^2\alpha = \pm\sin h^2\beta$.

22. Find the sum of the series

$$\tan^{-1}\frac{x}{1+2x^2} + \tan^{-1}\frac{x}{1+2\cdot 3x^2} + \ldots.$$

$$+ \tan^{-1}\frac{n}{1+n(n+1)n^2}.$$

23. Sum to the infinity the series :

$$\cos\alpha + \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cdot\frac{3}{4}\cos(\alpha+2\beta) + ...$$

24. If $\log \sin(\theta + i\phi) = L + iB$, prove that $2e^{2L} = \cos h\,\phi - \cos 2\theta$.
