MAY 2022 **U/ID 32502/UCM1B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write the expansion of $\tan\theta$ interms of θ.

2. Expand $\cos^4\theta$ interms of $\cos\theta$.

3. Write the expansion of $\sin n\theta$ interms of $\sin\theta$ and $\cos\theta$.

4. Write the expansion of $\cos 5\theta$ interms of $\sin\theta$ and $\cos\theta$.

5. Write the expansion of

   (a) $\sinh(x+y)$;

   (b) $\cosh(x+y)$.

6. Show that $\cosh 2x = \dfrac{1-\tanh^2 x}{1+\tanh^2 x}$.

7. Find $\sinh^{-1} x$ interms of x.

8. Find $\log(1+i)$.

9. Find inverse of the exponential function 2^{3x-5}.

10. Find $Log(4+3i)$.

11. Calculate the sum of $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin[\alpha+(n-1)\beta]$.

12. Find the sum of $\cos\alpha + \cos 5\alpha + \cos 9\alpha + \cdots$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that $\cos^8\theta = \frac{1}{12^8}\{\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35\}$.

14. If α, β, γ are roots of the equation $x^3 + px^2 + qx + p = 0$ prove that $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\pi$ radians, except in one particular case.

15. Show that $\frac{\sin 8\theta}{\sin\theta} = 128\cos^7\theta - 192\cos^5\theta + 80\cos^3\theta - 8\cos\theta$.

16. Separate into real and imaginary part of $\tanh(x+iy)$.

17. If $\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$, prove that $\cos 2\theta \cosh 2\theta = 3$.

18. If $(\alpha + i\beta) = b^{x+iy}$ prove that $y/x = \dfrac{2\tan^{-1}\beta/\alpha}{\log(\alpha^2 + \beta^2)}$ [λ only principal values are considered].

19. Find the sum of the series $\sum_{n=1}^{\infty} \dfrac{\sin n\theta}{2^n}$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Expand $\cos^7\theta \sin^3\theta$ in series of sines of multiples of θ.

21. Determine a, b, c such that $\underset{x\to 0}{Lt} \dfrac{\theta(a + b\cos\theta) - c\sin\theta}{\theta^5} = 1$

22. If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, show that

(a) $\theta = \dfrac{n\pi}{2} + \dfrac{\pi}{4}$.

(b) $\phi = \dfrac{1}{2}\log\tan\left(\dfrac{\pi}{2} + \dfrac{\alpha}{2}\right)$.

23. Find the general value of $(x+iy)^{\alpha+i\beta}$ and show that the sum of the moduli of the values, less than unity of $(1+i)^{(1+i)}$ is $\frac{e^{3\pi/4}}{\sqrt{2}}\cos ec\pi$.

24. Sum to n terms of the series $\cot^{-1}(2a^{-1}+a)+\cot^{-}(2a^{-1}+3a)+\cot^{-1}(2a^{-1}+6a)+\cdots$ and deduce the sum to ∞.