MAY 2023 **U/ID 32502/UCM1B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write the expansion of $\cos^n \theta$ interms of $\cos\theta$.

2. Expand $\sin^5 \theta$ interms of $\sin\theta$.

3. Expand $\tan^5 \theta$ interms of $\tan\theta$.

4. Write the expansion of $\cos 4\theta$ interms of $\cos\theta$ and $\sin\theta$.

5. Express $\cos\theta$ in ascending powers of θ.

6. Prove that $\cosh^2 x - \sinh^2 x = 1$.

7. Prove that $\sin ix = i \sinh x$.

8. Show that $\tanh 2A = \dfrac{2\tanh A}{1+\tanh^2 A}$.

9. Find the general logarithm of $\sqrt{3} + i$.

10. Find the principal logarithm of $\cos(x + iy)$.

11. Prove that $i^i = e^{-(4n+1)\pi/2}$, where n is any integers.

12. Sum to n terms of the series.

$\sin\alpha\sin 2\alpha + \sin 2\alpha\sin 3\alpha + \sin 3\alpha\sin 4\alpha + ...$

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that $2^8\cos^9\theta = \cos 9\theta + 9\cos 7\theta + 36\cos 5\theta + 84\cos 3\theta + 12\cos\theta$.

14. Show that $\frac{\sin 6\theta}{\sin\theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$.

15. Evaluate $\underset{x\to 0}{Lt}\ \frac{\tan 2x - 2\tan x}{x^3}$.

16. Expand $\cosh^8\theta$ interms of hyperbolic cosines of θ.

17. Show that $\log\tan(\pi/4 + ix/2) = i\tan^{-1}(\sinh x)$.

18. Sum to infinity of the series $\sum_{n=0}^{\infty}\frac{r^n\cos n\theta}{n!}$.

19. Sum to n terms of the series

$\tan\theta\sec 2\theta + \tan 2\theta\sec 2^2\theta + \tan 2^2\theta\sec 2^3\theta + ...$

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that the equation

$ah\sec\theta - bk\cos ec\theta = a^2 - b^2$ has four roots and that the sum of the values of θ which satisfy it is equal to an odd multiple of π radians.

21. Prove that

$64\left(\cos^8\theta + \sin^8\theta\right) = \cos 8\theta + 28\cos 4\theta + 35$.

22. If $\tan(\theta + i\varphi) = \cos\alpha + \sin\alpha$, show that

(a) $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$

(b) $\varphi = 1/2 \log \tan(\pi/2 + \alpha/2)$.

23. If $p^{\alpha+i\beta} = (x+iy)^{m+in}$ and only the principal values are considered, show that

(a) $\alpha = \frac{m}{2}\log_p\left(x^2+y^2\right) - n\tan^{-1}\left(\frac{y}{x}\right)\log_p e$.

(b) $\log\left(x^2+y^2\right) = \frac{2(\alpha m + \beta n)}{m^2+n^2}$.

24. Prove that $1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} + \ldots\ldots = \frac{\pi}{2\sqrt{2}}$.