MAY 2024 **U/ID 32502/UCM1B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write $\sin 4\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$.

2. If 'n' is an integer, what is the value of $\sin n\pi$?

3. If $x = \cos\theta + i\sin\theta$. Find the value of $x^4 - \frac{1}{x^4} = ?$

4. Prove that $\cos^5\theta = \frac{1}{16}[\cos 5\theta + 5\cos 3\theta + 10\cos\theta]$.

5. $\sinh 2x = 2 \sinh x \cosh x$.

6. Prove that $\cosh^{-1} x = \log\left[x + \sqrt{x^2 - 1}\right]$.

7. Separate into real and imaginary parts of $\cos(x + iy)$.

8. Prove that $\sin ix = i \sinh x$.

9. Find $\log(1 - i)$.

10. Find $\log(-2)$.

11. Write down Gregory's series.

12. Simplify $\frac{\sin h\alpha}{1!}+\frac{\sin h2\alpha}{2!}+\ldots+\infty$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Expand $\cos 6\theta$ in series of cosines of multiples of θ.

14. Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$.

15. Separate into real and imaginary parts of $\tanh(1+i)$.

16. Find the sum of 'n' terms of the series
$\operatorname{cosec}\theta\ \operatorname{cosec}\ 2\theta+\operatorname{cosec}\ 2\theta\operatorname{cosec}\ 3\theta+\operatorname{cosec}\ 3\theta\operatorname{cosec}\ 4\theta+\cdots$

17. Find $\log(1-i)$.

18. Sum to infinity the series
$$c\sin\alpha+\frac{c^2}{2!}\sin 2\alpha+\frac{c^3}{3!}\sin 3\alpha+\ldots$$

19. If $\cosh\ u=\sec\theta$, show that $u=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Expand $\sin^3\theta\cos^5\theta$ in a series of sines of multiples of θ.

21. If $\cos(x+iy) = \cos\theta + i\sin\theta$, prove that $\cos 2x + \cos 2y = 2$.

22. Find the sum of the series

$$\tan^{-1}\frac{x}{1+1.2x^2} + \tan^{-1}\frac{x}{1+2.3x^2} + \ldots + \tan^{-1}\frac{x}{1+n(n+1)x^2}$$

23. Find the sum to infinity of the series

$$\sin\alpha + c\sin(\alpha+\beta) + \frac{c^2}{2}\sin(\alpha+2\beta) + \ldots \text{ when } |c| < 1.$$

24. If $\log\sin(\theta+i\phi) = L + iB$ prove that $2e^{2t} = \cosh\phi - \cos 2\theta$.

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