DECEMBER 2022 **U/ID 32504/UCM2A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. If $y = (ax + b)^m$ then find y_n

2. If $xy = ax^2 + \frac{b}{x}$ then prove that $x^2 \frac{d^2y}{dx^2} + 2\left(x\frac{dy}{dx} - y\right) = 0$

3. If $x^3 + y^3 + 3axy$ then find $\frac{dy}{dx}$

4. Write the condition that is $f(x, y)$ attains its maximum at (a, b).

5. Find the envelope of the family of circles $(x - a)^2 + y^2 = 2a$, where a is the parameter.

6. Write the Cartesian formula for the radius of curvature.

7. Write the p-r equating of a curve.

8. Find the angle at which the radius vector cuts the curve $\frac{l}{r} = 1 + e\cos\theta$.

9. Write the formula for angle of intersection of two curves where equations are given in polar coordinates.

10. Write the formula for polar sub tangent and polar sub normal.

11. Find the asymptotes of $x^3 + y^3 = 3axy$.

12. Write the character of the asymptotes of a cubic curve.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find y_n where $y = \frac{3}{(x+1)(2x-1)}$

14. Find $\frac{du}{dx}$ when $u = x^2 + y^2$ where $y = \frac{1-x}{x}$.

15. Explain the lagrange's method of undetermined multipliers.

16. Find the envelop of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ where the parameters are related by the equations $a^2+b^2=c^2$ where c is a Constant.

17. Find the co-ordinates of the centre of curvature of the curve $xy=2$ at the point $(2,1)$.

18. Find the angle of intersection of the cardioids $r=a(1+\cos\theta)$ and $r=b(1-\cos\theta)$.

19. Find the asymptotes of $x^3+2x^2y-4xy^2-8y^3-4x+8y=1$

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. If $y=\sin\left(m\sin^{-1}x\right)$ prove that $(1-x^2)y_2-xy_1+m^2y=0$ and $(1-x^2)y_{n+2}-(2x+1)xy_{n+1}+\left(m^2-n^2\right)y_n=0$

21. Find the maximum and minimum values of $2\left(x^2-y^2\right)-x^4+y^4$.

22. From the polar equation of the parabola, show the $p^2=ar$.

23. Find the scope of the tangent which the initial line for the cardioid $r = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{6}$.

24. Find the asymptotes of

$$(x-y)^2(x-2y)(x-3y) - 2a(x^3 - y^3) - 2a^2(x+y)(x-2y) = 0$$
