DECEMBER 2023 **U/ID 32504/UCM2A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Find y_n, where $y = \frac{3}{(x+1)(2x-1)}$.

2. Find the n^{th} differential coefficient of $x^2 \log x$.

3. If $x^3 + y^3 + 3axy$, find $\frac{dy}{dx}$.

4. Write the formula for Taylor's expansion of $f(x, y)$.

5. Find the envelop of the family of circles, $(x-a)^2 + y^2 = 2a$, where 'a' is the parameter.

6. Write the Cartesian formula for the radius of curvature.

7. Write the formula for angle of intersection of two curves in polar coordinates.

8. Show that in the curve $r = e^{\theta \cot \alpha}$, the polar subnormal is $r \cot \alpha$.

9. The asymptotes of $x^2y^2 = c^2(x^2 + y^2)$ are the sides of a square.

10. Find the asymptotes of $x^3 + y^3 = 3axy$.

11. Find y_n when $y = \frac{1}{x^2 + a^2}$.

12. Find $\frac{du}{dx}$ where $u = x^2 + y^2$ where $y = \frac{1 - x}{x}$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. If $y = \sin(m \sin^{-1} x)$, prove that
$(1 - x^2)y_2 - xy_1 + m^2y = 0$ and
$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

14. If $xy = ae^x + be^{-x}$, prove that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$.

15. Explain the working rule for maxima and minima for $U = f(x, y)$.

16. From the polar equation of the parabola show that $p^2 = ar$.

17. For the cycloid $x = a(1 - \cos\theta),\ y = a(\theta + \sin\theta)$ find $\frac{dy}{dx}$.

18. Find the rectilinear asymptotes of $2x^4 - 5x^2y^2 + 3y^4 + 4x^3 - 6y^3 + x^2 + y^2 - 2xy + 1 = 0$.

19. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find the n^{th} differential coefficients of $\cos^5\theta \sin^7\theta$.

21. Transform $\frac{\partial^2 v}{dx^2} + \frac{\partial^2 v}{\partial y^2}$ into polar coordinates.

22. (a) Find the envelope of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{k^2 - a^2} = 1$, where '$a$' is the parameter.

(b) Find the envelope of the circles drawn on the radius vectors of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as diameter.

23. Find the angle of intersection of the cardioids, $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$.

24. Find the asymptotes of $(x-y)^2(x-2y)(x-3y)$

$-2a(x^3-y^3)-2a^2(x+y)(x-2y)=0$.
