DECEMBER 2024 **U/ID 32504/UCM2A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Find $\frac{d^n}{dx^n}[\log(ax+b)]$.

2. If $y = \frac{ax+b}{cx+d}$, find $\frac{d^2x}{dx^2}$.

3. If $y = xe^x$, find y_n.

4. Find $\frac{du}{dx}$ when $u = x^2 + y^2$ where $y = \frac{1-x}{x}$.

5. State Euler's theorem on homogeneous function.

6. State sufficient condition for maxima and minima.

7. Define curvature.

8. Find the radius of curvature of the curve $xy = c^2$ at (c,c).

9. Identify the curve $r\cos\theta = a$.

10. Find the slope of the curve $r = e^{\theta}$ at $\theta = 0$.

11. Write the rule of find asymptotes parallel to the axes.

12. Find the asymptote of $x^3 + y^3 = 3axy$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. If $y = a\cos 5x + b\sin 5x$, show that $\frac{d^2y}{dx^2} + 25y = 0$.

14. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2y$.

15. Transform $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$ into polar coordinates.

16. Find the centre of curvature of the ellipse.

17. Find the envelope of the family of straight lines $y = mx + \frac{3}{(2m)}$ where m is the parameter.

18. Find the angle at which the radius vector cuts the curve $\frac{1}{r} = 1 + e\cos\theta$.

19. Find rectilinear asymptotes of the curve $y^2(x^2 - y^2) - 2ay^3 + 2a^3x = 0$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If $y = (x + \sqrt{1 + x^2})^m$, prove that $(1 + x^2)\, y_{n+2} + (2n + 1)\, xy_{n+1} + (n^2 - m^2)\, y_n = 0$.

21. If $u = a^3x^2 + b^3y^2 + c^3z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the minimum value of y.

22. Find the equation of the evolute of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

23. Find the angle of intersection of the curves $r = a(\sin\theta + \cos\theta)$ and $r = a\cos 2\theta$.

24. Show that the asymptotes of cubic $x^2y - xy^2 + xy + y^2 + x - y = 0$ cut the curve again in three points which lies on the $x + y = 0$.

———————