MAY 2022 **U/ID 32504/UCM2A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. If $y = e^{ax}$ then find $\frac{d^2y}{dx^n}$.

2. If $xy = ae^x + be^{-x}$, prove that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$.

3. Write the Leibnitz formula for the n^{th} derivate of the product function *UV*.

4. If $u = f(x, y, z)$ and x, y, z are all functions of *t*, then write the total differential of *u*.

5. Write the condition that is $f(x, y)$ attain its minimum at (a, b).

6. What is the curvature of a circle of radius *r*?

7. What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1)?

8. Define the evolute of a curve.

9. Write the formula for angle between the radius vector and the tangent.

10. Show that in the curve $r = a\theta$, the polar sub tangent varies as the square of the radius vector and the polar subnormal is constant.

11. Define the term asymptotes.

12. Find the asymptotes of $(x + y)(x - y)(x - 2y - 4) = 0$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find the n^{th} differential coefficient of $\cos x . \cos 2x . \cos 3x$.

14. If $z = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.

15. Explain the working rule to examine a function $u = f(x, y)$ for maxima and minima.

16. Find the envelope of the family of straight lines $y + tx = 29t + 9t^3$, the parameter being t.

17. Show that the evolutes of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another Cycloid.

18. Find $\frac{dS}{d\theta}$ and $\frac{dS}{dr}$ for the cardioids $r = a(1 + \cos\theta)$.

19. Find the rectilinear asymptotes of the curve

$y^2(x^2 - y^2) - 2ay^3 + 2a^3x = 0$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Find the n^{th} differential coefficient of $x^2 \log x$.

21. Discuss the maxima and minima of the function $x^3y^2(6 - x - y)$.

22. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ is $\frac{a^n r^{-n+1}}{n+1}$.

23. Show that in the curve $r = e^{\theta \cot\alpha}$, the polar sub tangent in $r\tan\alpha$ and polar subnormal is $r\cos\alpha$.

24. Show that the asymptotes of the cubic $x^2y - xy^2 + xy + y^2 + x - y = 0$ cut the curve again in three ponies which lie on the line $x + y = 0$.

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