MAY 2023 **U/ID 32504/UCM2A**

Time : Three hours Maximum : 75 marks

PART A — (10×2 = 20 marks)

Answer any TEN questions

1. If $y = (ax + b)^m$ Find y_n.

2. Write the Leibnitz Formula for n^{th} derivative of a product of variables.

3. Find $\frac{du}{dx}$ when $u = x^2 + y^2$ where $y = \frac{1-x}{x}$.

4. State Euler's theorem.

5. What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1)?

6. Find the envelope of the family of circles $(x\,a)^2 + y^2 = 20a$. Where a is the parameter.

7. Find the slope of the tangent with the initial line for the cardioid $r = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{6}$.

8. Show that in the curve $r = a\theta$, the polar subtangent varies as the square of the radius vector and the polar subnormal is constant.

9. Find the asymptotes of $x^3 + y^3 = 3axy$.

10. Define asymptote.

11. If $U = x^3 + y^3 + z^3 + 3xyz$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u.$

12. Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point (2,1).

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions

13. Find the n^{th} differential coefficient of $\cos x \ \cos 2x \ \cos 3x$.

14. If $Z = f(x, y)$ and $x = r\cos\theta,\ y = r\sin\theta$ Prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

15. Find the maximum or minimum values of $2(x^2 - y^2) - x^4 + y^4$.

16. Prove that the $(P-r)$ equation of the cardioid. $r = a(1-\cos\theta)$ is $P^2 = \frac{r^3}{2a}$.

17. Find $\frac{ds}{d\theta}$ and $\frac{ds}{dr}$ for the cardioid $r = a(1+\cos\theta)$.

18. Show that the asymptotes of the cubic

$x^2y - xy^2 + xy + y^2 + x - y = 0$,

cut the curve again in three points which lie on the line $x + y = 0$.

19. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ is $\frac{a^n \, r^{n+1}}{n+1}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

20. (a) Prove that if $y = \sin(m \sin^{-1} x)$ $(1+x^2)y^2 - xy_1 + m^2 y = 0$.

(b) If $x = \sin\theta, \; y = \cos p\theta$, prove that $(1-x^2)y_2 - xy_1 + p^2 y = 0$

21. Find the evolute of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

22. Transform $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}$ into polar coordinates.

23. Find the angle at which the radius vector cut the curve $\frac{1}{r}=1+e\cos\theta$.

24. Determine the asymptotes of the curve $4(x^4+y^4)-17x^2y^2-4x\,(4y^2-x^2)+2(x^2-2)=0$ and show that they pass through the points of intersection of the curve with the ellipse $x^2+4y^2=4$.