MAY 2024 **U/ID 32504/UCM2A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find Y_n, where $Y = \dfrac{3}{(x+1)(2x-1)}$.

2. Write the Leibnitz formula for n^{th} derivative of a product.

3. Find $\dfrac{du}{dx}$, when $U = X^2 + Y^2$, where $Y = \dfrac{1-X}{X}$.

4. If $X^3 + Y^3 + 3aXY$, find $\dfrac{dy}{dx}$.

5. Write the Cartesian formula for the radius of curvature.

6. Find the envelope of the family of circles $(X-a)^2 + Y^2 = 2a$, where a is a parameter.

7. Write the formula for angle of intersection of two curves in polar coordinates.

8. Find the slope of the tangent with the initial line for the cardioid.

$r = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{6}$.

9. The asymptotes of $X^2Y^2 = C^2(X^2 + Y^2)$ are the sides of a square.

10. Define – asymptote.

11. Find Y_n when $Y = \frac{1}{X^2 + a^2}$.

12. If $U = X^3 + Y^3 + Z^3 + 3XYZ$, prove that

$X\frac{\partial u}{\partial x} + Y\frac{\partial u}{\partial y} + Z\frac{\partial u}{\partial z} = 3u$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find the n^{th} differential coefficient of $\cos X \ \cos 2X \ \cos 3X$.

14. If $XY = ae^X + be^{-X}$, prove that

$X\frac{d^2Y}{dX^2} + 2\frac{dY}{dX} - XY = 0$.

15. Find the maximum or minimum value of $2(X^2 - Y^2) - X^4 + Y^4$.

16. From the polar equation of the parabola show that $p^2 = ar$.

17. Find $\frac{ds}{d\theta}$ and $\frac{ds}{dr}$ for the cardioid $r = a(1+\cos\theta)$.

18. Find the rectilinear asymptotes of

$$2x^4 - 5x^2y^2 + 3y^4 + 4x^3 - 6y^3 + x^2 + y^2 - 2xy + 1 = 0.$$

19. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ is $\frac{a^n r^{-n+1}}{n+1}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find the n^{th} differential coefficients of $\cos^5\theta \sin^7\theta$.

21. Discuss the Maxima and Minima of the function $X^3Y^2(6 - X - Y)$.

22. Find the envolute of the ellipse $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$.

23. Find the angle at which the radius vector cut the curve $\frac{1}{r} = 1 + e\cos\theta$.

24. Find the asymptotes of

$$(X - Y^2)(X - 2Y)(X - 3Y) - 2a(X^3 - Y^3) - 2a^2(X + Y)(X - 2) = 0.$$

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