**MAY 2025** **U/ID 32504/UCM2A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. If $y = \frac{\log x}{x}$ find $\frac{d^2y}{dx^2}$.

2. Write the $n^{th}$ derivative of $y = \frac{1}{ax + b}$.

3. State Leibnit $z$ theorem.

4. If $u = x^2 + y^2 + z^2, x = e^t, y = e^t \sin t$ and $z = e^t \cos t$ find $\frac{du}{dt}$.

5. Find the envelope of the family of circles $(x - a)^2 + y^2 = 2a$, where $a$ is the parameter.

6. Define radius of curvature.

7. What is equation of circle of curvature?

8. Define – Maximum.

9. Write the formula for angle between the radius vector and the tangent.

10. Find the slope of $r = e^{\theta}$ at $\theta = 0$.

11. Find the asymptotes of $x^2y^2 = (a + y)^2(b^2 - y^2)$.

12. State the particular cases of intersection of a curve with its asymptotes.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. If $xy = ae^x + be^{-x}$, prove that $\frac{xd^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$.

14. Find $y_n$ where $y = \frac{3}{(x+1)(2x-1)}$.

15. Verify Euler's theorem when $u = x^3 - 3x^2y + 3xy^2 + y^3$.

16. Find the minimum value of $x^2 + y^2 + z^2$, when $x + y + z = 39$.

17. Find the radius of curvature at any point $(x, y)$ of the curve $\left(\frac{x}{a}\right)^{1/2} + \left(\frac{y}{b}\right)^{1/2} = 1$.

18. Find $\frac{ds}{d\theta}$ and $\frac{ds}{dr}$ for the cardiod $r = a(1 + \cos\theta)$.

19. Find the asymptotes of $(x - y)^2(x - 2y)(x - 3y) - 2a(x^3 - y^3) - 2a^2(x + y)(x - 2y) = 0$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find the $n^{th}$ differential coefficient of $\cos^5\theta \sin 7\theta$.

21. Find the extreme values of $a^3x^2 + b^3y^2 + c^3z^2$ subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1, a, b, c > 0$.

22. Prove that the evolute of the rectangular hyperbola $xy = c^2$ is $(x + y)^{2/3} + (x - y)^{2/3} = (4c)^{2/3}$.

23. Find the angle of intersection of the curves $r = a(\sin\theta + \cos\theta)$ and $r = a\cos 2\theta$.

24. Find the rectilinear asympotes of

$$2x^4 - 5x^2y^2 + 3y^4 + 4x^3 - 6y^3 + x^2 + y^2 - 2xy + 1 = 0$$

---