(6 pages)

DECEMBER 2023 **U/ID 32505/UCM2B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define Conjugate diameter.

2. Prove that the tangents at the ends of a pair of conjugate diameters of an ellipse form a parallelogram of constant area.

3. Write the equation of asymptotes of the hyperbola.

4. Find the equation of the circle which has its centre at the point (6, 1) and touches the straight line $5x + 12y = 3$.

5. Obtain the equation of the plane through the point (x_1, y_1, z_1) and parallel to the plane $ax + by + cz + d = 0$.

6. Find the distance between the parallel planes $x + 2y - 2z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$.

7. Find the Co-ordinates of the point of intersection of the line $\frac{x+1}{1} = \frac{y+3}{3} = \frac{z-2}{-2}$ with the plane $3x + 4y + 5z = 5$.

8. Prove that the equations to the line through (α, β, γ) at right angles to the lines $\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}$; $\frac{x}{l_2} = \frac{y}{m_2} = \frac{z}{n_2}$ are $\frac{x-\alpha}{m_1 n_2 - m_2 n_1} = \frac{y-\beta}{n_1 l_2 - n_2 l_1} = \frac{z-\gamma}{l_1 m_2 - l_2 m_1}$.

9. Find the centre and the radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y + 2z - 3 = 0$.

10. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z - 6 = 0$ at $(1,2,3)$.

11. Show that the plane $lx + my + nz = p$ will touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. If $(ul + vm + wn + p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$.

12. Define Orthogonal projection on a plane.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touch $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$ Find the locus of their poles.

14. Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a \cos\theta$.

15. The plane $lx + my = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α. Prove that the equation of the plane in its new position is $lx + my \pm z\sqrt{l^2 + m^2} \tan \alpha = 0$.

16. If the axes are rectangular and if l_1, m_1, n_1, l_2, m_2, n_2 are direction cosines. Show that the equation to the planes through the lines which bisect the angle between $\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}; \frac{x}{l_2} = \frac{y}{m_2} = \frac{z}{n_2}$ and at right angles to the plane containing them are $(l_1 \pm l_2)\, x + (m_1 \pm m_2)\, y + (n_1 \pm n_2)\, z = 0$.

17. Find the equation of sphere having its center at (1, 2, 3) and passing through (4, –3, 5) of the sphere and its passes through A (4, –3, 5).

18. Find the center and radius of the circle $x^2 + y^2 + z^2 - 2x + 4y + 2z - 6 = 0$:
$x + 2y + 2z - 4 = 0$.

19. Find the shortest distance between the axis of z and the line $ax + by + cz + d = 0$, $a_1 x + b_1 y + c_1 z + d_1 = 0$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose center is C meets the circle $x^2 + y^2 = a^2 + b^2$ at Q and Q'. Prove that CQ and CQ' are conjugate diameters of the ellipse.

21. Find the equation of the line joining the points $\left(2, \frac{\pi}{3}\right)$ and $\left(3, \frac{\pi}{6}\right)$. Deduce that the point $\left(2, \frac{\pi}{3}\right), \left(3, \frac{\pi}{6}\right)$ and $\left(\frac{6}{3\sqrt{3-2}}, \frac{\pi}{2}\right)$ are collinear.

22. A point P moves on a fixed plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. The plane through P perpendicular to OP meets the axes in A, B, C. The planes through A, B, C parallel to Co-ordinates planes interest in Q. Show that the locus of Q is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}$.

23. Find the magnitude and the equation of the line of shortest distance between the lines.

(a) $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$

(b) $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

24. Show that the locus of the centre of sphere of a co-axial system is a straight line.

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