DECEMBER 2024 **U/ID 32505/UCM2B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the centre of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$.

2. Find the polar of the focus in the directoriex, coordinate is $(ae, 0)$.

3. Find the condition in order that the lines $\frac{l}{r} = A\cos\theta + B\sin\theta$ may be a tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$.

4. Identify the conic

   $x^2 - 6xy + y^2 - 8x - 8y - 20 = 0$.

5. What is the equation of the circle in polar co-ordinates?

6. Find the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$.

7. Find the equation of the plane passing through (1, 1, 0) (1, 2, 1) and (–2, 2, –1).

8. Find the direction cosines of the lines $\frac{2x+1}{3} = \frac{4y-3}{1} = \frac{2z-3}{0}$.

9. Find the equation of the straight line joining the points (1, 2, 4) and (3, 5, 8).

10. Define Skew lines.

11. Find the equation of the sphere with centre (1, –1, 2) and radius 3.

12. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 2y - 4z - 5 = 0$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. In the ellipse $3x^2 + 7y^2 = 21$, find the equation of the equi conjugate diameters and their lengths.

14. Show that the locus of the poles with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ of tangents to the circles on the line joining the foci as diameter is the conic $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2 + b^2}$.

15. Find the asymptotes of the hyperbola

$3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

16. Find the co-ordinate of the foot of the perpendicular drawn the origin to the plane $2x - 3y + z - 7 = 0$.

17. Find the bisector of the acute angle between the planes, $3x + 4y - 5z + 1 = 0$, $5x + 12y - 13z = 0$.

18. Show that the lines $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and $\frac{x-5}{2} = \frac{y-8}{3} = \frac{z-7}{2}$ are coplanar and find the equation of the plane containing them.

19. Prove that the two spheres

$S_1 = x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$

$S_2 = x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$ touch each other and find the point of contact.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Obtain the equation to the chord joining the points $P[a \sec\theta, b \tan\theta]$ and $Q[a \sec\varphi, b \tan\varphi]$ in the form $\frac{x}{a}\cos\left(\frac{\theta-\varphi}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta+\varphi}{2}\right) = \cos\left(\frac{\theta+\varphi}{2}\right)$.

21. Trace the conic $\frac{2}{r} = 1 + \cos\theta + \sin\theta$.

22. A moving plane passes through a fixed point (α, β, γ) and intersects the coordinate axes at A, B, C show that the locus of the centroid of the ΔABC is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.

23. Find the shortest distance and the equation of the line of shortest distance in symmetrical form of the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

24. Find the equations of tangent planes of the sphere $x^2 + y^2 + z^2 - 4x - 4y - 4z + 10 = 0$ which are parallel to the plane $x - z = 0$.

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