MAY 2023 **U/ID 32505/UCM2B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Prove that the sum of the squares of two conjugate semi diameters of an ellipse is constant.

2. Find the co-ordinates of the point of intersection of tangents at the point where $x + 4y = 14$ meets the circle $x^2 + y^2 - 2x + 3y = 5$.

3. Write down the equation of conic.

4. Write the asymptotes of the hyperbola.

5. Find the equation of the plane passing through the intersection of the planes $x + y + z = 6$, $2x + 3y + 4z + 5 = 0$.

6. Prove that the equation $2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$ represents a pair of planes.

7. Find the equation of the line through the point $(1, 2, 3)$ parallel to the line $x - y + 2z = 5$; $3x + y + z = 6$.

8. Examine the nature of intersection of the planes $2x - 5y + z = 3 : x + y + 4z = 5 : x + 3y + 6z = 1$.

9. Find the equation of the sphere whose centre is $(-6, 1, 3)$ and radius 4.

10. Obtain the equation of the circle lying on sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 3 = 0$ and having its centre at $(2, 3, -4)$.

11. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point $(1, 2, 3)$.

12. Find the distance between the parallel planes $x + 2y - 3z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Show that the conjugate lines through a focus of an ellipse are at right angles.

14. Find the equation of the line joining two points $P(r_1, \theta_1)$ and $Q(r_2, \theta_2)$.

15. Find the equation to a plane through $P(a, b, c)$ and perpendicular to *OP*, where *O* is the origin.

16. Find the equation to the plane which passes through the point (x_1, y_1, z_1) and the line $(x-a)/l = (y-b)/m = (z-c)/n$.

17. If the equation of sphere is $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ prove that the centre is $(-u, -v, -w)$ and radius is $\sqrt{u^2 + v^2 + w^2 - d}$.

18. Prove that the tangent plane at $T(x_1, y_1, z_1)$ on the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ is $xx_1 + yy_1 + zz_1 + u(x + x_1) + v(y + y_1) + w(z + z_1) + d = 0$

19. Find the conditions that two given straight lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_1}{n_2}$ are coplanar.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that the acute angle between two conjugate diameters of an ellipse is minimum when they are equal.

21. Find the coordinates of the root of the perpendicular from the pole on the line joining the points (r_1, θ_1) and (r_2, θ_2).

22. Show that the origin lies in the acute angle between the planes $x+2y+2z=9$ and $4x-3y+12z+13=0$. Find the planes bisecting the angles between them and point out which bisects the acute angle.

23. Show that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$, $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect and find the co-ordinates of the point of intersection.

24. Show that the locus of the centre of spheres of a co-axal system is a straight line.

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